Modular Forms and Representation Theory
Spring 2023 MATH 514

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1 Overview

This is an introductory course to modular forms and its connection with the modern theory of automorphic representations. The focus of this course is on the basic notions and the correspondence between classical and modern objects. The course provides some working knowledge on automorphic representations to number theorists.

Even though some representation theory will be involved, no a priori knowledge of it is needed. We will develop these notions from scratch. The Lie group course nicely complements the content here. But strictly speaking there is not much overlap.

Learning objectives: After taking this course, students should have a good sense what automorphic representations are and how they connect to the classical notion of modular forms. They are able to start to study modern literature on automorphic representations.

2 Tentative syllabus and schedule

1. Holomorphic modular forms and Maass wave forms (2 weeks).
2. Representations of $GL_2(\mathbb{R})$ and $GL_2(\mathbb{Q}_p)$ (4 weeks).
3. Tate thesis (2 weeks).
4. Automorphic representations of $GL_2(\mathbb{A}_{\mathbb{Q}})$ (4 weeks).
5. Relation between modular forms and automorphic representations (2 weeks).
6. If time permits, we will discuss certain aspects of Langlands functoriality.

3 Prerequisites

1. Standard first year graduate course in analysis and algebra.
2. Some basic complex analysis (contour integrals, residue theorem, etc.)
3. Some basic topology (compactness, completeness, connectedness etc.)

4 Reference

1. Bump, D. Automorphic Forms and Representations.

2. Goldfeld, D. Automorphic Representations and L-Functions for the General Linear Group: Volume 1