SPECTRAL THEORY AND QUANTUM MECHANICS—Spring 2023 topics course proposal

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With the spectacular progress of quantum physics, sometimes dubbed Second Quantum Revolution, there comes a growing need for mathematicians and physicists to join their forces and work together in the fields where new discoveries are being made: open quantum systems, quantum information processing, measurement and control of quantum systems—to mention a few. This requires mutual understanding between the two communities. Mathematicians need to understand the basic concepts of quantum theory, and physicists need to know the language and methods of relevant mathematics—in particular spectral theory of operators in Hilbert spaces. This course will discuss the mathematical concepts and results in their natural physical context. Its main content will be spectral theory, including theory of unbounded operators. Methods from real, complex and harmonic analysis will be used, but no advanced knowledge is assumed. Mathematics or Applied Mathematics core analysis and familiarity with basic Hilbert space theory prepares a student to learn physical concepts in the mathematical language. On the other hand, a physics student with a good background in quantum mechanics will be prepared to study the mathematical approach to the field. I am planning to discuss in detail the following topics:

- Basic theory of bounded and unbounded operators in a Hilbert space

- Spectral theorem

- Stone-von Neumann theorem—representation of unitary groups of operators

- Weyl theorem on the essential spectrum

- Mathematics of the hydrogen atom

- Compact operators and their role in spectral analysis

- Trace-class operators and density operators. Introduction to quantum statistical mechanics

- Rayleigh-Ritz principle. Variational methods in quantum theory

- Continuous spectrum and RAGE theorem

- Wave operators and introduction to scattering theory

- Introduction to open quantum systems

Course objective and learning goals: The objective of the course is to discuss mathematical foundations of quantum theory and to show the mathematics, that grew out of physical applications. Mathematics students will one of the most important chapters of analysis in the context of the exciting field of quantum theory. Physics students will learn the mathematical methods important for progress in the subject and a new perspective on their field. The second objective of the course is to bring mathematics and physics students together, make them talk and exchange languages and ways of thinking. I will encourage participation of students and class discussions.

Literature: For the main part of the course, I will use the book "A mathematical companion to quantum mechanics" by Sternberg [1]. It grew out of a similar course, taught by the author for many years at Harvard. It is a very hands-on book which from the beginning does interesting analysis and mathematical physics, without lengthy introductory chapters. I am planning to use other books occasionally, in particular the Reed and Simon classic "Methods of modern mathematical physics" [2]. I will also discuss some themes from the excellent physics book "Exploring the quantum" by Haroche and Raimond [3], to introduce the newer and rapidly growing field of open quantum systems and to see a recent Nobel prize winner's perspective on modern quantum physics.

1. S. Sternberg, "A mathematical companion to quantum mechanics", Dover 2019
2. M. Reed, B. Simon, "Methods of modern mathematical physics", Academic Press
3. S. Haroche, J.-M. Raimond, "Exploring the quantum: atoms, cavities and photons", Oxford 2006