Title: Optimal transport and its applications

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Text: Self-generated notes with supplemental references: Villani's "Optimal transport old and new," Ambrosio and Gigli's "A user's guide to optimal transport", Santambrogio's "Optimal transport for applied mathematicians"

Description of topics to be covered: Optimal transport has its origins in Napoleonic France and, roughly, the problem of efficiently distributing resources. In recent years, it has become a powerful tool in an impressively wide variety of fields in pure and applied mathematics, from PDE to machine learning. This course will cover its main ideas as well as their application in PDE, numerical analysis and beyond. This will include the Wasserstein metric, duality, Benamou-Brenier formula, gradient flows and the JKO scheme, etc. Students will present a paper in their field of interest using optimal transport as a final project.

Prerequisites: measure theory and L^p spaces

Learning outcomes: Deep understanding of the Wasserstein metric and the ability to apply it and the main ideas outlined above in the context of the student's field of interest

Approximate schedule:

* Weeks 1-3: Monge and Kantorovich problems, duality
* Weeks 4-5: Wasserstein space
* Weeks 6-10: Gradient flows, Benamou-Brenier formula, JKO scheme
* Weeks 11-14: Applications (e.g., existence for nonlinear PDE, numerical schemes, image processing, machine learning)
* Week 14-15: Student presentations