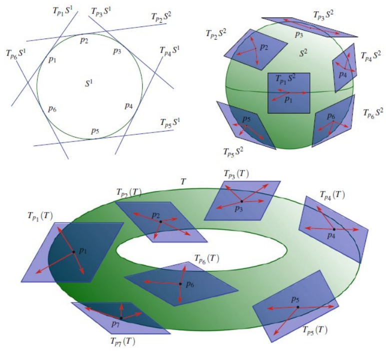
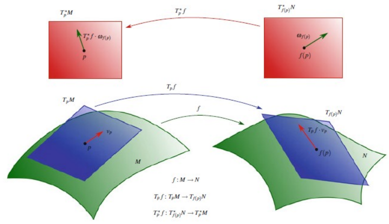
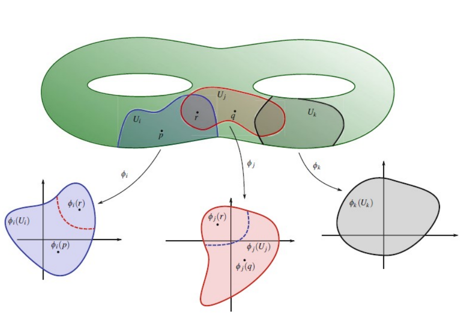

Math 496T Spring 2024
Differential Forms and Calculus on Manifolds

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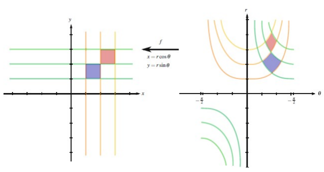
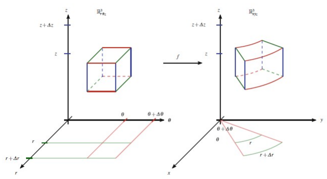
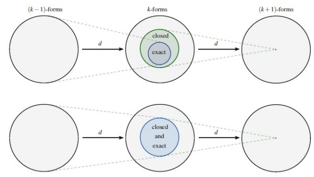
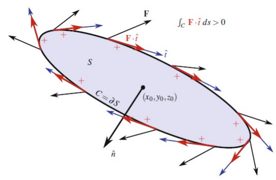

About the Course: This course dovetails nicely with vector calculus and is meant to help students transition from calculus to more advanced mathematical topics and more abstract ways of thinking, as well as to explore vector calculus from a deeper theoretical perspective. The main topic in this class, differential forms, play an important role throughout both theoretical mathematics and physics. (In particular, differential forms provide a beautiful way to write down Maxwell's equations.) Unfortunately, they are usually not covered well in most undergraduate curriculums, yet an understanding of them is needed in many graduate-level classes. Therefore, this class should also be of interest to any math major who hopes to go to graduate school.

Requirements: Vector calculus (linear algebra will be helpful but not necessary.)

Objectives: Students should learn the basic ideas behind differential forms, and how differential forms can be used to explain standard vector calculus concepts. Calculus on manifolds will be discussed, with R^2 and R^3 being the primary manifold examples. Tangent and cotangent spaces and bundles, the wedge product, exterior differentiation, pull-backs and push-forwards, integration of differential forms, and the generalized Stokes' theorem will be covered. The student will be able to relate these more theoretical ideas to concepts they already know from vector calculus such as div, grad, and curl, as well as the fundamental theorem of line integrals, Stokes' theorem, and the Divergence theorem. This provides students with a firm grounding on calculus on manifolds necessary for a range of more advanced topics. Additionally, this class helps provide students a good "bridge" between more concrete mathematics topics, like calculus, and more abstract and theoretical concepts and ways of thinking.

$$\begin{tikzcd}
C(\mathbb{R}^3) \arrow[r, "\nabla", "\text{grad}"'] \arrow[d, "\text{id}"] & T\mathbb{R}^3 \arrow[r, "\nabla\times", "\text{curl}"'] \arrow[d, "\flat"] & T\mathbb{R}^3 \arrow[r, "\nabla\cdot", "\text{div}"'] \arrow[d, "*\circ\flat"] & C(\mathbb{R}^3) \arrow[d, "*"] \\
\bigwedge^0(\mathbb{R}^3) \arrow[r, "d"] & \bigwedge^1(\mathbb{R}^3) \arrow[r, "d"] & \bigwedge^2(\mathbb{R}^3) \arrow[r, "d"] & \bigwedge^3(\mathbb{R}^3)
\end{tikzcd}$$

Text: *A Visual Introduction to Differential Forms and Calculus on Manifolds*, Fortney (Birkhauser)