OPERATOR ALGEBRAS AND QUANTUM THEORY—SPRING 2025 TOPICS COURSE PROPOSAL

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Outline

Observables in quantum theory are represented by operators in a Hilbert space. These operators can be added, multiplied by scalars and, less obviously, they can be composed. They thus form an *operator algebra*. This is the reason why theory of operator algebras—in addition to being a beautiful part of functional analysis—plays a fundamental role in the mathematics of quantum theory. This course will be an elementary introduction to operator algebras, in particular to von Neumann algebras, most important for applications. My aim is to introduce the basic language and foundations of the theory, assuming no more than familiarity with a Hilbert space—and enthusiasm—and reach the point when the theory can be applied to physical systems. While the main content of the course will be mathematical, the physical applications and illustrations will be the main motivation. The tentative list of topics is:

1. Bounded operators in a Hilbert space
2. Spectral theorem
3. C^*-algebras and von Neumann algebras
4. Commutative Banach algebras
5. Classification of von Neumann algebras
6. Commutant theory
7. Trace theory and states on von Neumann algebras
8. Physical examples

I plan to follow 2009 Berkeley lectures by Vaughan Jones as closely as possible.

Additional material will be selected as needed from two monographs on the subject:

S.-V. Stratila and L. Zsido: Lectures on von Neumann algebras. Cambridge University Press 2019.

O. Bratteli and D. Robinson: Operator algebras and quantum statistical mechanics, vol. I. Springer 1987.

Learning outcomes: The students will gain a working knowledge of the theory of von Neumann algebras and its role in quantum theory, including spectral theory, quantum probability and mathematical representation of states of quantum systems.