Gaussian Measures (Spring 2027)

Instructor: I. Fatkullin

Gaussian measures lie at the foundation of modern probability, statistics, mathematical physics, and data science. Because they accurately model systems driven by many independent factors, understanding their structure and properties provides the essential tools needed to solve a large variety of theoretical and practical problems.

In this course, we will study the core features of these measures, progressing from the one-dimensional case to finite-dimensional spaces and finally concentrating on the infinite-dimensional setting.

Approximate Schedule

The course will be divided roughly into three parts:

I. Finite-Dimensional Gaussian Measures (January)

We will study the structure and fundamental properties of finite-dimensional Gaussian distributions, including concepts such as covariance matrices, generating functions, higher-order moments and cumulants, and the Wick–Isserlis theorem.

II. Construction of Infinite-Dimensional Measures (February – March)

We will transition to infinite-dimensional Hilbert and Banach spaces and discuss classical constructions such as abstract Wiener measure and the Cameron–Martin space. Connections to related concepts in mathematical physics, including Gelfand triples, rigged Hilbert spaces, and functional integration, will be illuminated.

III. Additional Special Topics (April)

As time permits, we will peek into additional topics such as the Wiener–Khinchin theorem, the Karhunen–Loève theorem, and Malliavin calculus. Other suggestions by the students will be considered.

Textbook

I will draw some of the content from D. W. Stroock's *Gaussian Measures in Finite and Infinite Dimensions*. This book makes full use of advanced analysis, however, so I will adapt the lectures to better match the mathematical background of the class.

Prerequisites

A solid foundation in undergraduate linear algebra, real analysis, functions of complex variables, and probability is required. Prior exposure to functional analysis would be helpful but is not essential; the necessary advanced topics will be introduced in class.

Learning Outcomes

By the end of the course, students should be able to:

- Understand the core theory underlying the construction of Gaussian measures on standard finite- and infinite-dimensional spaces.
- Perform calculations involving finite-dimensional distributions, generating functions, moments, and cumulants.
- Develop a solid conceptual understanding of infinite-dimensional Gaussian measures, the Cameron–Martin space, and abstract Wiener spaces.

Evaluation

I will assign homework related to the lectures, including exercises such as completing calculations and proving lemmas. Students who submit solutions will receive feedback.

The course grade will be based on the quality of the final project, consisting of a paper and a presentation. Students may select a topic from a list that I will provide or choose a topic related to their own research, provided that it is relevant to the course.