Sunflower, Spirals, and the Cross Ratio

A self-guided exploration — take it at your own pace

You are about to explore two interactive sunflower tools. You can dive in freely — there are no wrong moves — or follow the steps below. The questions are there if you want them; skip any that feel like too much.

The tools: ① **Sunflower Cross Ratio Explorer (Sunflower_Cross_Ratio_Explorer.html)**

② **Geometric CR at Each Seed. (Geometric_CR_at_Each_Seed.html)**

Part A — First look, and the Fibonacci numbers (tool ①)

Open the Sunflower Cross Ratio Explorer and spend a minute just looking before touching anything.

1. What do you notice first?

Diamonds? Spirals? Something else? Try following one spiral with your finger from the centre outward.

- ✦ How many spirals can you count going one way? Going the other?
- ✦ Do the spirals look the same near the centre as near the edge?

2. Turn on the spiral families

In the Layers panel, tick 21 spirals (gold) and 34 spirals (blue). Toggle them on and off as often as you like, then add 13 (purple) and 55 (green).

- ✦ Are those numbers familiar?

> ***Tip:*** *1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 … each number is the sum of the two before it. These are the Fibonacci numbers, and a sunflower almost always has two consecutive ones as its spiral counts.*

- ✦ Turn on 89 and 144 as well. Where in the flower is each family easiest to see?
- ✦ Notice that 13 + 21 = 34, and 21 + 34 = 55. Does that show up anywhere in the picture?

3. Zoom

Use the zoom slider to move between the centre and the rim.

- ✦ Near the rim, which two families fit the pattern best? Near the centre?
- ✦ The best pair changes as you move outward. Can you find roughly where it switches?

Part B — Building a pencil of lines (tool ①)

Four lines through one point are called a pencil. The cross ratio is a single number measuring how such four lines sit relative to one another.

4. Anchor the pencil

Click any seed. That seed becomes the centre of the pencil. Now click a few nearby seeds — each click draws a line from the centre through that seed. The cross ratio appears once you have four lines.

- ✦ What happens to the number if you pick four seeds at random?
- ✦ Now clear, and try choosing your four seeds along the spiral directions instead. Does the number change?

5. Use Auto

Click Clear, click a seed, then press Auto: show 2 spiral families + bisectors.

- ✦ What value does the cross ratio take?
- ✦ Try it at several different seeds, near the centre and near the rim. Does it ever change?

> ***Tip:*** *Auto builds the four lines as: the two spiral directions, and the two bisectors of the angle between them. The inner and outer bisectors of any angle are always harmonic with respect to the two sides — so this construction gives CR = −1 (equivalently 2) every single time, by geometry alone.*

6. The additive pencil — and the Forsythe fold

Click Clear, click a seed, then press Add L+R and L+2R (additive pencil).

- ✦ Only two directions were chosen. Where did the other two lines come from?

> ***Tip:*** *If the two spiral directions are L and R, this button adds L+R and L+2R. The new lines are sums of the old ones — exactly the additive step 13 + 21 = 34 that generates the Fibonacci numbers.*

This is the same mechanism the choreographer William Forsythe uses when he folds an arm: the upper arm gives one line, the forearm another, and the fold creates new lines that are combinations of the first two. The sunflower and the dancer are running the same arithmetic — one in seeds, one in limbs.

- ✦ If a fold of the arm makes a new line from two old ones, what in the sunflower plays the role of the elbow?

7. Anharmonic group

Look at the sidebar panel listing six values: λ, $1/\lambda$, $1-\lambda$, $1/(1-\lambda)$, $\lambda/(\lambda-1)$, $(\lambda-1)/\lambda$.

- ✦ When the cross ratio is harmonic, those six collapse to just three: -1, 2, $\frac{1}{2}$. Why might one configuration give several numbers?

Part C — The honest measurement (tool ②)

Everything in Part B used exact directions, so the answer came out exactly harmonic by construction. Tool ② instead measures the pencil from the actual positions of real neighbouring seeds.

8. Read the field of numbers

Open Geometric CR at Each Seed. With the mode set to 8 nearest seeds (best 4), every seed is labelled with its own measured cross ratio.

- ✦ What values do you see? Are they exactly 2?
- ✦ Use the colour scale at the left. Where are the seeds closest to 2? Where are they furthest?

> ***Tip:*** *The sunflower is not a perfect mathematical object. It grows one seed at a time, and the cross ratio comes out at about 1.98 or 1.99 rather than exactly 2 — close, and getting closer as the flower grows.*

9. Hover a seed

Hover over any seed to see the four lines actually used, drawn to four real neighbouring seeds.

- ✦ Do those four lines look harmonic to you?
- ✦ Compare a seed near the centre with one near the rim. Which pencil looks more symmetric?

10. Change the Fibonacci pair

Switch from Auto to a fixed pair — F13 + F21, then F21 + F34, then F34 + F55.

- ✦ Each fixed pair works well in one ring and poorly elsewhere. Can you see the rings?
- ✦ Compare those rings with the spiral families you turned on in Part A. Do the boundaries line up?

11. More seeds

Raise the seeds slider to 800 or more.

- ✦ Do new spiral families appear at the rim?
- ✦ Do the measured cross ratios get closer to 2, or further away?

Part D — Stepping back

- ✦ The same number turns up in the sunflower, in a dancer's arms, and in the geometry of parallel lines meeting at a vanishing point. Why might one number appear in all three?
- ✦ The Fibonacci numbers come from adding the two before. The Forsythe fold makes a new line by adding two old ones. Is that the same fact twice, or two different facts?
- ✦ A sunflower knows nothing about cross ratios — it is packing seeds as densely as it can. What does it mean for a mathematical fact to be "hidden" in a growing plant?
- ✦ The measured value is 1.99, not 2. Is the sunflower approximately harmonic, or is 2 the number it is heading towards?