Math 223

Disclaimer:
It is not a good idea to rely exclusively on reading through old exam solutions as a way to prepare for the final exam. In particular, this semester's course director may not have written any of the exams available from this page, so the ones he/she gives will almost certainly have a somewhat different flavor.

Topic Warning:
Because the topics taught differ slightly from semester to semester, it is not a good idea to use the old exams to gauge the content of the exams this semester.

1. (8) Consider two vectors $\vec{v} = 4\vec{i} - \vec{j} + a\vec{k}$ and $\vec{w} = a\vec{i} + 5\vec{j} - \vec{k}$. For what values of a are $\vec{v}$ and $\vec{w}$ perpendicular?

2. (8) Find an equation for the plane passing through the points $(0, 2, 1)$, $(1, 1, 5)$, and $(2, 0, 11)$.

3. (8) Find a vector of magnitude 10 normal to the plane $5x + 3y = 6z + 1$.

4. (8) At which point (or points) on the ellipsoid $x^2 + 4y^2 + z^2 = 9$ is the tangent plane parallel to the plane $z = 0$?

5. (8) Find a parametric equation for a line through the point $(1, -3, 5)$ and parallel to the vector $5\vec{i} + 3\vec{j} - \vec{k}$

6. (8) Find the directional derivative of $f(x, y) = x^2y + y^2x$ at the point $(1, -1)$ in the direction of $3\vec{i} - 4\vec{j}$.

7. (7) Compute the flux integral $\int_S \vec{F} \cdot \overrightarrow{dA}$ where $\vec{F} = \vec{i} + \vec{j} - \vec{k}$ and S is the surface $z = x^2 - y^2, \quad 0 \le x \le 3, \quad 0 \le y \le 3$, oriented upwards.

8. (20) Consider the integral $\int_0^1 \int_{(8y)^{1/3}}^{2} \frac{1}{1+x^4} dx dy$.

   (a) Interchange the order of integration. Show your work by including a sketch of the region of integration.

   (b) Evaluate the integral.

9. (20) Let $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$. Evaluate the following:

(a) $\int_C \vec{F} \cdot \vec{dr}$ where C is the line from $(0,0,0)$ to $(1,1,1)$.

(b) $\int_S \vec{F} \cdot \vec{dA}$ where S is the triangle in the plane $y = 10$ with vertices $(0,10,0)$, $(4,10,0)$, and $(0,10,1)$, oriented in the direction of increasing y.

(c) $\int_S \vec{F} \cdot \vec{dA}$ where S is the sphere of radius 2 centered at $(5,5,0)$, oriented outward.

10. (15) Let $H(x, y, z) = \sin(2x + y) + z$. Find the equation of the tangent plane to the level surface $H(x, y, z) = 5$ at the point $(\pi, \pi, 5)$.

11. (20) Let $\vec{F} = (y+z)x\,\vec{i} + y\,\vec{j} + xyz\,\vec{k}$.

(a) Find $\text{curl}(\vec{F})$.

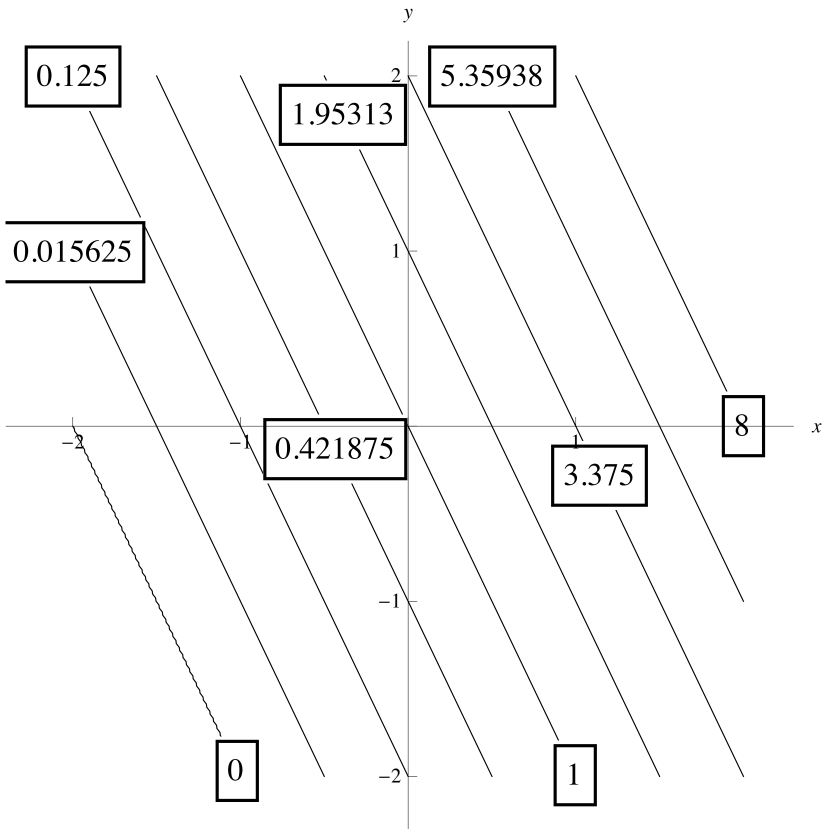
(b) Let S be the surface $x^2 + y^2 + z = 25$, with $0 \leq z \leq 25$, oriented upward. Find the value of the flux integral $\int_S \text{curl}(\vec{F}) \cdot \overrightarrow{dA}$.

12. (15) Let a be a constant, $a \neq 2$, and consider function $f(x, y) = \frac{1}{2}x^2 + 2y + 2xy + ay^2$.

(a) Find the critical point of f.

(b) Find all values of a so that the critical point is a global minimum.

13. (20) Consider the contour diagram for the function $f(x, y)$ sketched below.



(a) Sketch a graph of $f(x, 0)$.

(b) Determine whether the following quantities are positive, negative, or equal to zero.

$f_{xx}(0, 0)$ is ______ $f_{xy}(0, 0)$ is ______

(c) If *all contour lines* are parallel to the line $2x + y = 0$, then determine the direction in which the gradient of f points, *as a unit vector*.

14. (15) Rewrite the integral $\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{0}\int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} xy\ dz\ dx\ dy$ in spherical and cylindrical coordinates.

(a) In spherical coordinates, use the order of integration $d\rho\ d\theta\ d\phi$.

(b) In cylindrical coordinates, use the order of integration $dz\ dr\ d\theta$.

15. (20) Consider the 2-dimensional force field $\vec{F} = 2xe^{x^2-5y}\,\vec{i} - 5e^{x^2-5y}\,\vec{j}$.

(a) Is $\vec{F}$ conservative? If so, find a potential function $f(x, y)$ whose gradient is $\vec{F}$.

(b) Find the work done by the force field $\vec{F}$ in moving an object from $P(0, 2)$ to $Q(-2, 0)$ along the path formed by C_1 followed by C_2 as shown in the figure below. C_1 and C_2 may be parametrized as follows:

$$C_1: \quad x = t, \quad y = 2 - t, \quad 0 \le t \le 2,$$
$$C_2: \quad x = 2\cos t, \quad y = -2\sin t, \quad 0 \le t \le \pi\ .$$