Math 223

Disclaimer:
It is not a good idea to rely exclusively on reading through old exam solutions as a way to prepare for the final exam. In particular, this semester's course director may not have written any of the exams available from this page, so the ones he/she gives will almost certainly have a somewhat different flavor.

Topic Warning:
Because the topics taught differ slightly from semester to semester, it is not a good idea to use the old exams to gauge the content of the exams this semester.

1. (30) Solve the following problems. No partial credit.

   (a) If $\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}$, and $\vec{v} = a\vec{i} + 2\vec{j} - 4\vec{k}$ (a is a constant), then $\vec{u} \cdot \vec{v}$ is equal to

   A. $a - 14$.

   B. $2(a - 7)$

   C. $2a - 10$

   D. $a + 10$

   E. 7

   (b) Find a normal vector for the plane $7y = z$.

   (c) Find $\dfrac{\partial f}{\partial y}$, where $f(x, y, z) = y^2 e^{xyz}$. Simplify your answer as much as possible, factoring where possible.

(d) If S is the plane $x = 0$ oriented in the positive x direction, then the surface area element $\overrightarrow{dA}$ is which one of the following quantities?

A. $\vec{j}\,dxdz$.

B. $\vec{k}\,dydz$

C. $-\vec{i}\,dydz$

D. $\vec{i}\,dydz$

E. $-\vec{k}\,dydx$

(e) The figure below shows the contour diagram of which function $f(x, y)$?

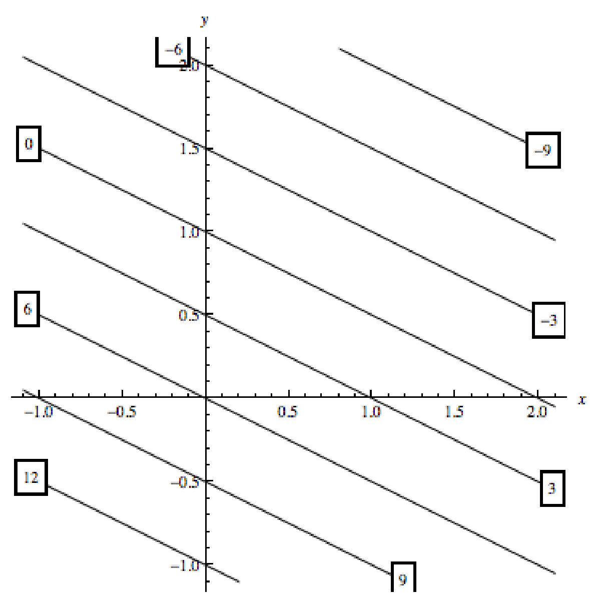
A. $f(x, y) = 6y - 3x + 6$

B. $f(x, y) = \frac{1-x}{2}$

C. $f(x, y) = e^{-3x-6y+6}$

D. $f(x, y) = -3x - 6y$

E. None of the above.

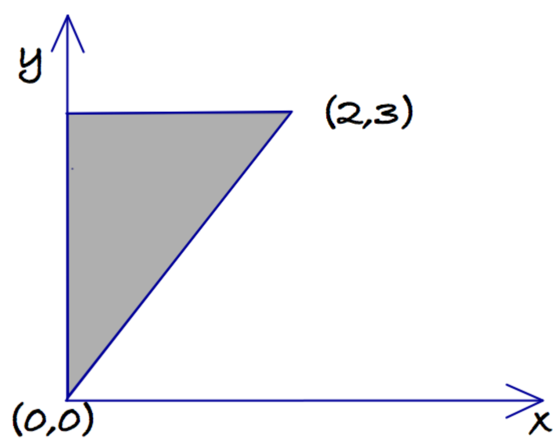


(f) Let R be the two dimensional region shown in the figure below. What is $\int_R f(x,y)dA$?

A. $\int_0^3 \int_0^{2y/3} f(x,y)dxdy$ **B.** $\int_0^3 \int_0^{3x/2} f(x,y)dxdy$

C. $\int_0^3 \int_0^{2y/3} f(x,y)dydx$ **D.** $\int_0^2 \int_0^{3x/2} f(x,y)dydx$

E. $\int_0^3 \int_{3x/2}^{2} f(x,y)dxdy$



2. (20) Consider the integral $\int_{-1}^{0} \int_{0}^{y^2} e^{x/y^2} \, dx dy$.

   (a) Interchange the order of integration. Show your work, including a sketch of the region of integration.

   (b) Evaluate the original integral. Give an exact answer.

3. (30) Let $\vec{F} = y\,\vec{i} + 2z\,\vec{j} + (1-z)\,\vec{k}$. Evaluate the following:

(a) $\int_C \vec{F} \cdot \vec{dr}$ where C is the straight line from the origin to $(1, 3, 1)$.

(b) $\int_S \vec{F} \cdot \vec{dA}$ where S is the rectangle $0 \le x \le 2$, $1 \le y \le 4$, $z = 0$, oriented upwards.

(c) $\int_S \vec{F} \cdot \vec{dA}$ where S is the sphere of radius 2 centered at the origin, oriented outwards.

4. (20) Let $f(x,y) = \sqrt{1+4x+y^2}$, and let P be the point $(1,2)$.

(a) At P, what is the direction of maximal increase for the function f? Give your answer as a unit vector.

(b) Find the directional derivative of f at P in the direction of $3\vec{i} - 4\vec{j}$.

5. (15) Let $H(x, y, z) = x^2 + y^2 + 2z^2$, and let S be the level surface $H(x, y, z) = 4$. Find the coordinates of a point P on the surface S where the tangent plane to S is parallel to the plane $2x + 4z = 0$.

6. (20) Suppose S is the surface obtained by taking the union of the upper hemisphere of a sphere of radius 2 centered at $(0, 0, 4)$,

$$S_1 = \left\{(x, y, z) \text{ such that } x^2 + y^2 + (z-4)^2 = 4 \text{ and } z \geq 4\right\}$$

and an open cylinder of radius 2 centered around the z axis,

$$S_2 = \left\{(x, y, z) \text{ such that } 0 \leq z \leq 4 \text{ and } x^2 + y^2 = 4\right\} \ .$$

The orientation of S is away from the origin.

a.) Sketch the surface S.

b.) Evaluate the integral $\int_S \left(\text{curl}(\vec{F})\right) \cdot \overrightarrow{dA}$, if $\vec{F}$ is the vector field $\vec{F} = y\,\vec{i} - x\,\vec{j} + xy\,\vec{k}$.
Hint: It is strongly recommended to use Stokes' theorem to simplify the surface integral.

7. (20) Consider the cubic polynomial $f(x,y) = \frac{5}{2}x^2 - xy + 15x + \frac{1}{75}y^3 - 3y$.

(a) Find the critical point(s) of $f(x,y)$.

(b) Use the second derivative test to classify, if possible, the critical point(s) you have found.

8. (15) Let S be the paraboloid $x^2 + y^2 + z = R^2$, $0 < z \leq R^2$, oriented upward, and let $\vec{F} = x\,\vec{i} + y\,\vec{j} + z^2\,\vec{k}$. Find the flux of the vector field $\vec{F}$ through the surface S.

9. (20) Consider the 2-dimensional force field $\vec{F} = (4e^{-2x} + 3y^3)\vec{i} + 9xy^2\vec{j}$.

a) Is $\vec{F}$ conservative? If so, find a potential function $f(x, y)$ whose gradient is $\vec{F}$.

b) Find the work done by the force field $\vec{F}$ in moving an object from $P(0, 1)$ to $Q(1, 2)$ along the path $y = 1 + \sin(\pi x/2)$ from $x = 0$ to $x = 1$.

10. (10) An asteroid is a cylindrical mass of ice, 100 km tall and with radius 5 km. The density of the asteroid varies linearly along its long dimension, varying from zero at one end to $10kg/m^3$ at the other. Set up a triple integral representing the total mass of the asteroid.