Learning Mathematics with adult learners:
Drawing from a parents’ perspective

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Abstract

In this paper we discuss some examples of how adults learn mathematics within the context of parents’ workshops. We look at mathematics education as a reflexive field that includes learners’ experiences as ways to explore, learn, and teach mathematics. Findings show that learners (parents) prefer contextualized activities, as well as concrete problems, rather than abstract algorithms. Situated learning and egalitarian spaces to promote dialogue among participants are key components in the pedagogical practice with adult learners. Our data show that parents’ attitude towards some forms of participation have their roots in their personal schooling experiences. Data analysis also illustrates that dialogue allows parents to share their own ways to do mathematics, learning one from each other. Workshops become zone of proximal development spaces that promote parents’ mathematics learning through their active participation. Workshops also become spaces for parents to build meaning to concrete mathematical concepts, grounded on their prior daily live experiences.

Introduction

In September 2007, a group of researchers met at Florence for a conference. Participants discussed topics such as the meaning of numeracy; different approaches to teaching mathematics to adult learners; and mathematics and popular culture. Several questions related to approaches for adult mathematics education arose, including, should we rely on the pattern of “lecture, sample problems, and homework” or should we focus on learners’ experiences as sources of meaningful activities? Do we have to privilege teacher authority inside the classroom, or learners are the ones who should take the lead? Is pedagogy transmission-based, or participative-constructed? What is the role of egalitarian dialogue? How can we include all participants’ voices inside the classroom?

1 Final conference of EMMA – European Network for Motivational Mathematics for Adults, September 2007, Florence.
http://www.statvoks.no/emma/index_program_final_florence.htm
practices? What kinds of barriers do teachers have to face when teaching mathematics to adults?

Research on mathematics education of adult learners draws attention to the idea that there are different approaches to teach and learn mathematics with adult people (FitzSimons, 2007). For adults, mathematics is not a “new” field of knowledge, but a plethora of experiences. Coben (2007) declares that mathematics and numeracy are not interchangeable terms. The transfer of knowledge from experiences grounded on “real world situations” to the formal domain of mathematics (“school mathematics”) has been a well explored field in the ALM (Adult Learning Mathematics) community. Knowles (1968) knew that adults are self-directing. They (we) are autonomous individuals with criteria to make decisions. Furthermore, Knowles demonstrated that adults are not tabulae rasae, but they (we) have experiences, which can play a crucial role in the learning process. Later on, Mezirow (1997) applied Habermas’ critical communicative theory to the adult education field. He argued that teachers working with adults should draw on arguments rather than power relations. Flecha (2000) goes further and declared that educative practices in adult education should be grounded in egalitarian dialogue among all participants. But, what does it mean in terms of strategies to teach mathematics to adults?

In this paper we address questions such as this and in general issues related to the pedagogical approaches to the teaching of mathematics to adult learners. We draw on data from a current study with parents and we build on prior work that we have done exploring issues related to pedagogy and content when working with parents (Civil, 1999, 2001, 2002, 2004). The next section presents the methodology that we used to collect and analyze the data; then we discuss several examples to illustrate elements of teaching and learning mathematics based our work with parents. We conclude with some implications to consider in our work with adults.

Methodology

This presentation is grounded on data from a study carried out with parents in Tucson, Arizona. Through parents’ voices we explore different ways to teach and to learn mathematics with adult learners. We build extensively on prior research with parents and mathematics (Civil, 1999, 2001, 2002, 2004; Civil & Andrade, 2003). We draw on situated learning episodes using a socio-cultural (Civil, 2002; González, Andrade, Civil, & Moll, 2001) and a dialogical perspective (Flecha, 2000). We aim to connect a cognitive-affective analysis of learning strategies used by adults to learn mathematics with a perspective grounded on the “adult learning mathematics” (ALM) research tradition, in order to highlight some of the key components that help adults learn mathematics.

Data come from mathematics workshops for parents whose educational level ranges from second grade to college degrees, with a median of sixth grade. Participants in those workshops were Latino parents, some of them recent immigrants from Mexico. We also take data from interviews that focus on their perception of teaching and learning
mathematics. The fieldwork was carried out in three different settings: two elementary schools and one middle school. We invited parents to participate in these workshops through members of the community and the school administration. Participants were encouraged to reflect on their perspective on learning and teaching mathematics through different activities.

The mathematics content of these workshops focused on the parents’ children’s school curricula and on topics of their personal interest. The timeframe for these workshops at the middle school spans from the fall of 2006 to the fall of 2007, and the interviews took place during the fall of 2007. The content of the workshops were packaged in modules of seven sessions each: Two modules were held during the first semester and one module per semester after that. Each session ran for 1.5 hours and its structure contained a brief introduction to the mathematical concept to be studied, some problems to be worked out individually and in small groups followed by a group discussion of the solution to the problems.

At the middle school, the content of the workshops changed, per parents’ request, from following the school curriculum to studying fractions. Another modification was the meeting time: from early in the morning, after the parents had taken their children to school, to the evening. Mothers who suggested this change argued that it was more likely that their husbands would be able to participate if the workshops were held after work hours.

At one elementary school workshops were held in different rounds from the fall of 2005 to the spring of 2007. The content of the workshops changed from one semester to another. At the beginning activities were grounded on mathematics as a critical issue, drawing from a social justice approach. Then, also per parents’ request, the content changed to activities based on the school reform-based curriculum. Researchers worked collaboratively with schoolteachers in order to prepare the activities. At the other elementary school workshops were carried out also in different rounds from the fall of 2005 to the fall of 2006. Activities also were based on the school curriculum as well as on parents’ request.

**Discussion**

We have organized our discussion along six topics that emerged from our data analysis: contextualization, experiencing mathematics, prior experience, critical thinking, sense-making, and affect. Each topic is illustrated with one vignette.

*Vignette 1. Learning Mathematics in (with) context*

Several parents were working on an activity regarding how to read a graph, as well as how to use it to represent (and to explain) a particular phenomenon in our world. José María (the facilitator) asked parents to form small groups to work on a story related to some aspect from their daily life experiences. Previously he had shown in the chalkboard an example based on how a friend of his uses his car during the week. He presented a
graph representing how much gasoline the car has from Monday to Sunday, day by day. Then everybody in the class discussed the story underlying to this line on the graph. Parents came up with some interpretations on how that person uses his car to go to work every day, and then uses his car to go away on the weekend, when he gets gasoline, an so forth. After that dialogue, the facilitator (José María) asked each group to think on something related to their daily life and to use the line graph in order to represent that activity and to explain it to the whole class. After about thirty minutes, one of the mothers (Luisa) came to the front of the classroom to present her group’s work (see Figure 1).

Figure 1. Line graph from Luisa’s group.

The connection to her daily life seemed quite natural to Luisa, and the connection between a line graph and the progression of a phenomenon over time was readily grasped. Luisa explained how that line represented the energy that somebody spends in a week. On the horizontal axis the group wrote the days of the week, from Sunday to Saturday. Then, on the vertical axis they represented the energy level, from less to more. Luisa narrates her group’s explanation in the next quote:

**Luisa**: Es el nivel de energía que tenemos a diario. O sea, hay veces que amanecemos con mucha energía, hay veces que amanecemos con menos (apuntando a los niveles de la gráfica del lunes y martes con una pluma). Hay veces que subimos, vamos subiendo (apuntando al punto alto de el miércoles) y bajando (bajando con la pluma en miércoles). El jueves amanecen uno más o menos, con menos energía (apuntando al jueves), el viernes con menos (señala baja del viernes con la pluma), el sábado va bajando la energía (señala con pluma) y el domingo pues estamos agotados (voltea a donde están los demás). Sí, y el lunes empezamos otra vez. Así, así lo entiendo yo.

*[That’s the level of energy we have daily. So, there are times when we wake up with*
a lot of energy, there are times when we wake up with less energy (pointing at the levels on the graph, on Monday and Tuesday with a pen). There are times when we go up, we are going up (pointing on the high point on Wednesday) and down (going down with the pen on Wednesday). On Thursday we wake up so-so, with less energy (pointing at Thursday), on Friday with less (she points going down on Friday with the pen), on Saturday energy is going down (points with the pen) and Sunday we are exhausted (turns to look at the class). Yes, and on Monday we start all over again, that is how I understand it. ]

(Luisa, Math For Parents workshop, Sept. 2006)

Luisa’s explanation captures the notions of slope and increasing/decreasing in an intuitive way, based on the steepness and direction she gave to each line segment in her graph. Thus, this can be used as a context to introduce notions such as axes, function, coordinates, slope, direction, decreasing and increasing functions. This is an example of what Lave and Wenger (1991) have called “situated learning.” They presented a study with adult people using diet to talk about mathematics and use them to solve real problems. Lave and Wenger learned about adults’ personal histories, drawing on adults’ experiences with grocery shopping and dieting. They found that adults used algorithms such as ratios, as well as arithmetic procedures and concepts like integers, decimals, fractions, addition, subtraction, multiplication, and so forth, when they went shopping. It was something natural for them. Regular activities (such as shopping) were the perfect frame to make sense of formal mathematics. Lave and Wenger concluded that learning occurs in situated environments. Adults go back to their daily life experiences to understand formal mathematics, putting them in context (thus context becomes part of the cognitive process of learning). Our data seem to be consistent with this analysis. Luisa also turned to her everyday knowledge to understand what that line graph meant.

Vignette 2. Experiencing Mathematics

Mathematics can be a very abstract body of knowledge disconnected from our direct experience with the real world. Plaza, González, Montero, and Rubio (2004) suggest that adults prefer “concrete mathematics” rather than an “abstract” one, and they choose ways to represent mathematics closer to their everyday experiences rather than algorithms or proofs. They prefer the former because it is easier for them to understand the meaning underlying a mathematical notion when it is connected to real examples, rather than to a collection of unfamiliar symbols written on the chalkboard. This next vignette takes us to a session where the facilitator was explaining the idea of median. It was the second module, which was focused on statistics. More specifically, the topics were data representations and data analysis. The following activity takes place during the Measures of central tendency lesson. The participants had already been exposed to the concept of the median by using this technique: Each participant takes one value from the data set; places oneself in order, in relation to the rest of the group; and the two people at the extremes of the line, one from each end point, separate from the line until only one or two are left. (The facilitator uses the metaphor of “going to dance” when each pair leaves the line.) If one person is left alone, the value he or she holds is the median; otherwise,
they have to add up the values of the couple left at the end, and divide the sum by two. We see this technique being enacted below as parents solve the problem of finding the median of the given average house price per school district in their city:

Marisol: ¿Cuánto tienes?
[How much do you have?]

Rigoberto: Ciento once.
[One hundred eleven.]

Marisol: ¡Ah no!
[Oh, no.]

Irene: Yo tengo ciento nueve.
[I have one hundred nine.]

Marisol: Yo [tengo] doscientos sesenta y siete; del más grande [señalando a la persona a su derecha] al más chiquito [mirando hacia la dirección opuesta].
[I have two hundred sixty-seven; from the largest (pointing to person to her right) to the smallest (looking towards the other direction).]

Luisa: Ciento cincuenta y tres.
[One hundred fifty-three]

Rigoberto: ¿Quién tiene el más chiquito?
[Who has the smallest one?]

Francisco: ¿Estamos hablando por cantidades o...?
[are we talking about quantities or...?]

Marisol: A ver, aquí es, es del más grande al más chiquito. Yo tengo doscientos sesenta y siete, ¿quién tiene más cerca [inaudible]?[Let’s see, this is from the largest to the smallest. I have two hundred and sixty-seven, who is closer to (inaudible)?]

Luisa: ¿Ciento cincuenta y tres? [Se coloca al lado izquierdo de Marisol.]
[One hundred and fifty-three? (moves to the left of Marisol.)]

Marisol: Ajá, ciento cincuenta y tres. ¿Y luego?
[Uh huh, one hundred fifty-three and then?]

Irene: Tengo ciento sesenta y uno.
[I have one hundred sixty-one.]

Marisol: No. ¿Y luego?
[no, and then?]

Rigoberto: Luego aquí.
[Then here.]

Marisol: Ciento once [leyendo el papel de Rigoberto], ¿Y tú?
[one hundred eleven (reading Rigoberto’s piece of paper), and you?]

Norma: Ciento veintitrés.
[one hundred twenty-three]

Marisol: Ciento veintitrés, ciento once,
[one hundred twenty-three, one hundred eleven.]

Irene: Ciento nueve.
[One hundred nine.]

Marisol: Entonces vas tú y luego va la señora... vas tú y luego va
[Then you go and then Ms..., you go and then]
Adelaida: No, ella tiene ciento nueve y yo tengo cien.  
[no, she has one hundred twenty and I have one hundred nine.]

Marisol: ¡Ah, OK! ¿Y tú?  
[ah, Ok, and you?]

Francisco: Sesenta y nueve.  
[Sixty-nine.]

Marisol: OK. Así estamos ya.  
[Ok, so we are ready then.]

As soon as the eight participants were ordered in line, the facilitator (José María) started to take them aside two by two. Finally only two persons were left (in the middle of the queue). At that point José María asked what to do when there is two individuals left. This was only the second time they were practicing the concept of the median. By enacting the process the first time, it seems that the participants were able to reproduce the technique at ease. A by-product of this implementation was the creation of an informal reference to the process of eliminating the extreme values of the ordered set, namely the metaphor to take them out to dance.

José María: OK. Muévanse un poquito más hacia allá [moviendo las manos], háganse un poquito más para allá; perfecto. Y comiencen a bailar. Salgan a bailar. Ahí van.  
[ok, move a little bit over there (motioning with his hands), a little bit over there, perfect, and start dancing; come dancing, here you go.]

Marisol: Y luego vas tú enfrente. Se va a quedar el señor  
[And then you go in front. Mr. M is going to be the one left]

José María: Se quedan dos. Aja. Muy bien… ¿Qué hacíamos cuando no nos quedaba un valor sino que había dos valores en el medio?  
[there are two left, aha, very well… What did we do when we had not just one value but two values in the middle]

Luisa: Se dividía.  
[We divided.]

José María: Se dividía. Exacto. Muy bien.  
[We divided, exactly, very well.]

Marisol: Se suma y se divide entre dos.  
[We add them and divide them by two.]

(Math For Parents workshop, Nov. 2006)

Parents were able to address the idea of median with an even number of frequencies, which is not an easy task since it includes three steps to solve it: first to order the data, second to split the data by half, and then to take the values in the middle, add them and divide them by two. We do not know for sure but wonder if the dance metaphor played a role in their understanding of how to find the median as it provided them with a concrete and visual approach. As Plaza et al. (2004) suggests, adults are more likely to prefer concrete and pragmatic manners to learn mathematics, rather than abstract ones.

There was one mother, Marisol, who took the lead organizing the work of the group. Marisol’s role in these workshops needs to be pointed out, as she is not one of the
participants per se. She works at that school and is the contact who recruited the parents to these workshops and who was instrumental in their attending (she would call them to remind them and even offer to pick them up if needed; she made sure that there were refreshments for the workshop; she basically took care of many of the logistics). Marisol was a mother participant in a prior project directed by the third author and, as a result of that project, she had already been facilitating mathematics parent workshops at the school prior to our work there. We want to point out that while Marisol took over the group’s discussions sometimes in this first series of workshops, she has since taken a more behind the scenes seat to allow the other parents to take a more active role.

**Vignette 3. Transfer of mathematical knowledge**

This session focused on the study of the slope. Parents were working with graphic representations, and the facilitator (José María) introduced the idea of slope through a practical example. He asked parents to measure different stairs in the neighborhood (the rise and the run of one step for each stair), then to write those measures in a paper and bring them to the next working session. Parents did their “homework” and they wrote different measures in their notebooks. Then, José María asked the parents which one was the hardest stair to climb, and which one was the easiest one. With this information he was ready to explain what does it mean slope and how to calculate it. However, rather than to write on the chalkboard the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, he said: You have to divide the length of the vertical line (rise) over the length of the horizontal one (run). After that a discussion about how to translate “rise” and “run” into Spanish came up, and then Marisol came back to the mathematical idea with these words:

**Marisol:** Yo pienso que ahí la vertical tiene un papel muy importante. Definitivamente. Porque sí es entre más es la que lleva... es, es la base de todo. Porque entre más altura, pues es mayor inclinación. . . Ahora me voy a empezar a fijar qué tantos son los escalones para no caminar [risas de fondo]. Porque si, porque entre más alto esté, ajá, porque entre más alto esté la, la vertical, pues va a ser más... la inclinación. Porque siempre va a llevar el predominante la división para sacar el rati... el, el, la... el número de la pendiente. Entonces siempre va a ser determinante.

[I think that the vertical there has a very important role. Definitely. Because if the more is the one that carries... it is, it is the bases for everything. Because the more the height, then the more the incline. Now I will start paying attention to how many steps [there are] so that I don’t walk (background laughs). Because yes, because the higher it is, uh-huh, the higher it is the, the vertical, then the more the incline will be. Because it is always the case that what it’s going to be predominant is the division to obtain the rati... the number of the slope. So, it will always be a determining fact.]

**Adelaida:** La horizontal es la que va a guiarte tantos escalones para que se te haga menos pesado.

[The horizontal is the one that is going to lead you many steps so that it turns less hard on you.]
Marisol: Porque acuérdate, si es la es... vamos a figurarnos que un escalón está así, ¿no? Date cuenta, así, como hiciste tú. Se hace una base para un desnivel para poner arreglos lo que sea. Si es chiquito, y esto es muy alto, pues va a ser más. Pero sí da así, da más margen, como más descanso, la parte, lo que es la parte horizontal. No, no... ¿sí? ¿Ya me entendieron? ¡Qué cantinflidad!

José María: Y entonces es ¿sólo vertical? ¿sólo horizontal? ¿O son los dos?

Una voz (a voice): Es la combinación.


José María: Para determinar la medida de la pendiente. ¿Y cómo se determina la medida de la pendiente, dijimos?

Varias voces: Vamos a dividir la...

Rigoberto: la altura por...

Esperanza: La vertical por la horizontal.

José María: Muy bien.

(Math For Parents workshop, Nov. 2006)

In this excerpt we can see several interesting aspects. Marisol was trying to understand the concept of slope making connections with her experience grounded in her knowledge of the real world. She assumes that slope depends on height, thus the more height, the more slope we can notice. We can hypothesize that Marisol was making a connection between her experience and the new mathematical concept that José María was introducing that session. This is a very common practice for adult learners. They are not tabulae rasaes; they have a repertoire of previous knowledge grounded on their life experiences (Knowles, 1968). At that point José María asked Marisol whether the slope depends on the vertical part only or on the horizontal part. Somebody answered that both mattered (“the combination”). Then Esperanza elaborated more on this idea. She said that

2 “Cantinflidad,” or rather “cantiñflada” refers to a style of speech used by the Mexican actor and comedian Mario Moreno “Cantinflas” (1911-1993) characterized by the use of many words in a convoluted way and vacuous in content.
we have to take into account both, vertical and horizontal, in order to determine the height of the slope. José María asked again how could we calculate the measure for the slope. He used the word measure rather height, which was the word used by Esperanza to talk about the slope. This word, altura (height), is a more common and intuitive word in than measure. In this context, height is not used in the same way as they would use it to refer to the height of a building, for example, but more as in comparison with the horizontal displacement. For this reason, we see her use of that specific word as transference from the real-world-based knowledge to the formal one. This insinuates that Esperanza has an understanding of what “slope” means in her everyday experience and she was using that knowledge to make sense to the formal meaning of slope (as well as Marisol did).

Vignette 4. Mathematics and critical thinking

On one hand, our work with parents is influenced by the kind of mathematics their children are learning in school. The three vignettes we just presented are an example of this approach. They were all based on similar activities from the middle school mathematics curriculum used at that school. On the other hand, our work with parents is also influenced by critical mathematics education, in particular as it applies to adult learners (Frankenstein, 1983; Greer, 2008). In this view, mathematics is a tool to read and write critically our world (Gutstein 2006; Gutstein & Peterson, 2005). Mathematics for social justice and mathematics for becoming a critical citizen (Skovsmose & Valero, 2001, 2002) are orientations that we find in adult education courses. We also included this orientation in our workshops. We present one example of this here. This example comes from our work at one of the elementary schools with a group of mothers. We used the activity “ten chairs of inequality” (Kellogg, 2005). This activity shows two images with a group of ten people and ten chairs. The first image shows the ten people with one chair for each person. The second image shows a person stretched over seven chairs, another person sitting on one chair, and the remaining eight people piled up on the two chairs left. Parents were asked to discuss these pictures. Here are their comments:

Tamara: Mmmmm... tienen que ser diez sillas aquí también, entonces están usando tres

[Mmmmn... there must be ten chairs here too, then they are using (three
Ruth: dos
[two
Tamara: Uno, dos, tres, cuatro, cinco, seis, siete, ...
[One, two, three, four, five, six, seven, ...
Ruth: ... ocho, nueve.
[... eight, nine.]
Tamara: Pues por eso.
[So, that’s the reason.]
Ruth: Por eso, pero esa está ahí sola, en, en... en dos sillas están ocho personas.
[That is why, but that one is there by herself, in, in, ... there are eight people in two chairs.]
Loli: Hay una correcta nomás, sentadita en una.
(otra voz dice: y diez sillas)
[There is only one proper one, sitting there, on one (chair) 
(another voice says: and ten chairs)]

**Javi (facilitator):** Entonces esto qué os sugiere, imaginarnos que en vez de personas las sillas fueran el dinero del mundo, y las personas las personas del mundo.

[So, what does this suggest to you? Imagine that instead of people, chairs were the world’s money, and the people were the world’s people.]

**Tamara:** ¡Aquí estamos nosotros! (señalando haciendo el movimiento de un círculo sobre el dibujo de la silla donde se sientan las ocho personas; risas)

[Here we are! (pointing and making a circle around the drawing of the chair with eight people; laughs).]

**Loli:** Hechos bolas. És la única rica. (señalando a la persona que está sola en varias sillas).

[All tangled up. She is the only rich one (pointing to the person who is alone occupying several chairs).]

**Tamara:** No, y estos de acá (señalando a los que están en las sillas del medio).

[No, and these ones here (pointing to those at the chairs in the middle).]

**Javi:** No, y este de aquí, éste tiene siete.

[No, and this one here, this one has seven.]

**Tamara:** Este es millonario.

[This one is a millionaire.]

**Loli:** Ah. (risas) ¿Cuáles?

[Ahh, (laughs) Which ones?]

As we can see in this excerpt, Tamara was very perceptive while judging this situation: The person lying down on the seven chairs was the “millionaire,” while the people piled up on two chairs represented “us” (probably bringing in her understanding of social class to make sense of that picture). Tamara was using her knowledge from her life experiences about the wealth distribution in the world to read this picture, and in doing so she was also using mathematical knowledge.

**Vignette 5. Making sense of mathematics through previous experience**

Adults’ everyday experiences are rich in terms of mathematics. Although they (we) do not use formal mathematics usually, it is very common that adult people use informal methods to solve problems and these methods often involve some sort of mathematical reasoning. This next vignette comes from a session where we were working on problem solving strategies with the same group of mothers as in Vignette 4. We presented them with this problem (the problem was given in Spanish):

Problem: John buys a watch for $60, and after that, he sold it to Paul for $80. One week later, John buys this watch from Paul for $100, and after that he sold it to Kevin for $120. How much has John earned?

The mothers engaged in a lively discussion about how much John had earned. Ruth, Elena, and Tamara were solving the problem together. Ruth played the role of the “more
capable peer” envisioned by Vygostky (1978). According to the Russian psychologist, learning is a process carried out both in context and collectively. Vygostky suggested that learners could go further (in their own process of learning) if they collaborate with other peers more capable. The ZPD is the potential that everybody is able to develop with the help of another person more capable. In this sequence Ruth became the more capable peer, while Tamara (as well as Elena) was trying to understand why John gets $40 dollars after he sold the watch to Kevin. Javi (the facilitator) also takes part in this excerpt; he mostly asked questions to encourage the conversation and to inquire about how they were thinking about the problem.

**Javi (facilitator):** O sea, John dice que compró un reloj por 60 dólares luego se la vende a Paul por 80 dólares. [John says that he bought a watch for 60 dollars and then later y sold it to Paul for 80 dollars.]

**Tamara:** Ahí gana veinte. [There he makes 20].

**Javi:** Pues pasaron una semana… ahmm, ahí ganó veinte. Se lo vuelve comprar a Paul por 100 dólares. [So a week passed… ahmm, there he made 20. He buys the watch back from Paul for 100 dollars.]

**Ruth:** Ahí perdió veinte. [There he lost 20.]

**Javi:** Ahí perdió veinte. [There he lost 20.]

**Tamara:** Mhmmm

**Javi:** Y luego se lo vende a Kevin por 120 dólares. [and later he sells it to Kevin for 120 dollars.]

**Tamara:** Volvió a recuperar los veinte. [He recuperates the twenty]

**Javi:** Ahmm entonces ¿cuánto ha ganado John? [so how much did john win?]

**Tamara:** 40.

**Javi:** ¿Cómo lo has hecho? [How did you do it?]

**Ruth:** Ya lo hice. [I already did it.]

(risas)

(Ruth: Bueno lo hice la otra clase. [well I did it in the other class.]

**Javi:** Pero ¿por qué cuarenta? [But why forty?]

**Ruth:** Pues aquí lo puse, mira cuando compré el reloj en 60 y luego se lo vende a Paul en 80 ganó veinte. Puse más veinte, ¿no? [Well, here I put, look when he buys the watch at 60 and later he sells it to Paul at 80, he wins twenty. I put plus twenty, right?]

**Tamara:** Veinte… ajá.
Ruth: Dice: pasada una semana se lo vuelve a comprar a Paul por 100 dólares.
[It says, a week passed and he sells it back to Paul for 100 dollars]
Elena: Ahí le quedan los veinte todavía.
[He is still left with twenty]
Ruth: Aquí tiene menos 40. Entonces él va perdiendo 20 dólares, tengo menos 20. Y luego dice “y luego se lo vende a Kevin por 120 dólares. ¿Cuánto ha ganado John? Como tenía... lo vendió en ciento veinte y tenía menos 20, lo resté, me quedaron cien.
[Here he has minus forty, so he continues losing 20 dollars, he has minus 20. And later it says “and later he sells it to Kevin for 120 dollars. How much did John gain? How he had. He sold it at 120 and he had minus 20, I subtracted and I had 100 left.]
Mmmmm
Ruth: Entonces le sumé, cien menos 60, ganó 40, cuando la compra inicial fue de 60.
[And so I added up, 100 minus 60, He gained 40, when he bought it initially for 60.]
Tamara: No….
[No…]
Javi: ¿Lo has entendido?
[Did you understand it?]
Tamara: No.

Ruth explained why John won $40 dollars after buying/selling the watch several times. Tamara was trying to understand the whole thing. But she had a hard time keeping track of the different transactions. The facilitator (Javi) asked Tamara if she understood the answer. Tamara answered “no.” The following dialogue ensued:

Elena: Según yo veinte, veinte lo que ha ganado nada más.
[According to my calculations I have twenty. Twenty is what he gained]
Tamara: John lo compró en 60.
[John bought it at 60.]
Tamara: Sí, lo compró en 60 y se lo vendió por 80.
[Yes, he bought it at 60 and he sold it at 80.]
Tamara: Y se lo vendió por… 80.
[He sold it for…. 80.]
Elena: Quedaron veinte de ganancia
[That leaves twenty in gains.]
Ruth: Veinte de ganancia
[Twenty in gains.]
Tamara: Luego…
[Then…]
Ruth: Y luego se lo vuelve a comprar por 100 dólares.
[And then he buys it back for 100 dollars.]
Elena: Pero todavía le quedan los ochenta que tenía, más los veinte, así que tiene 100, y lo compró en ciento veinte.
[But there is still the eighty that he had, plus the twenty, so he has 100, and he bought it at 120.]

**Elena:** Puso otros veinte de su bolsa.

**Ruth:** No pero llevaba, llevaba, en rojo llevaba veinte dólares; él, llevaba perdiendo veinte.

[No but he had. He was twenty dollars in red; he carried twenty in losses.]

**Elena:** Sí.

[Yes]

**Ruth:** Entonces, al venderlo en ciento veinte, menos veinte, le quedan cien dólares

[100 dollars and the twenty in profit.]

**Elena:** Cien dólares y los veinte de ganancia.

[100 dollars and the twenty in profit.]

**Ruth:** Cien. No pero le había costado sesenta, bueno según yo no estoy bien, no sé. (risas) Entonces ya ganó cuarenta

[100, No but it had cost him 60. Ok, maybe I’m not so sure, I don’t know. And so he profited 40.]

**Javi:** ¿Se les ocurre algún método para intentar averiguar...?

[Can you think of any method to attempt to find out?]

**Tamara:** No, pues no lo entiendo.

[No, well I don’t understand it.]

Now Elena gets involved in the discussion also. She was arguing with Ruth about how much John has earned after selling and buying and selling again the watch. Tamara was still listening and trying to follow the dialogue. Finally Tamara declared that she did not understand it yet. We argue that both “dialogue” (Flecha, 2000) and “participation” are components of the learning process. When (adult) learners are able to participate in the dialogues in a “learning situation,” this may point to an illustration of learning. We cannot say that participation always leads to effective learning, but when learners are engaged in the interaction, they also participate in the process of learning, and thus they participate actively in the process of creation of meaning. However, that evidence does not solve the question about the role played by the “internal talk” (Vygotsky, 1978). Are silent learners also engaged in the participation (and consequently in the process of learning), or are they not? Silent participation is another type of “active” participation, or not? Our data show that Tamara was first a silent observant, but then she takes an active role in the whole interaction as we will see in the excerpt below. But first we want to bring our attention to another important element: At some point Ruth said, “he was in red twenty dollars.” The important word here is red, because it shows that Ruth was making connections between the real world and formal mathematics. Ruth was talking about negative integers, which correspond to the idea of red number in the real world.³ This connection to the real world is common in adult education (as discussed in Vignettes 2 and 3). Adult learners usually go to their daily life experiences to find meaningful connections.
examples to connect with formal mathematics ideas. Going back to the watch problem, dialogue among participants followed in this way:

**Ruth**: Léelo.

[Read it.]

**Ruth**: Oh, yo lo pondré en el pizarra.

[Oh, I could put it on the chalkboard.]

**Tamara**: A ver... Yo compré un reloj y te lo vendí a ti en 80. Entonces yo me gané veinte.

[Lets see, I bought a watch and I sold it to you at 80. So I profited 20.]

**Elena**: Sí, porque lo compraste en 60.

[Yes because you bought it at 60.]

**Ruth**: Eso son 20 buenos.

[That’s 20 good.]

**Elena**: Lo compraste en 60 y lo vendiste en 80. Son veinte para ti.

[You bought it at 60 and you sold it at 80. That’s twenty for you.]

**Tamara**: Entonces vienes y me lo vendes, entonces, cien.

[So you come to me and you sell it to me, so, 100.]

**Elena**: Lo compraste en 60 y lo vendiste en 80. Son veinte para ti.

[You bought it at 60 and you sold it at 80. That’s twenty for you.]

**Tamara**: Tú ganaste veinte, tú veinte. Pero yo, pero yo he perdido.

[You profited 20, you 20. But I , but I lost.]

**Ruth**: Trae veinte malos, o sea, caiste en rojo pues.

[You bring twenty bad, that is, you fell into red then.]

**Tamara**: Menos veinte. Y luego…. Lo vende por 120, yo lo vendo por 120.

[Minus 20. And later... He sells it for 120, I sell it for 120.]

**Ruth**: Pero tienes veinte malos, te quedan cien buenos, y lo compraste en 60, a lo último ganas cuarenta.

[But you have 20 bad, you have 100 good left, and you bought it at 60, at the end you make 40.]

**Javi**: Mhmm.

**Ruth**: O sea a lo bueno, bueno te quedan 40.

[That the very best you have 40 left.]

Now Tamara is playing an active role in the session. Ruth asked Tamara to read the problem while she, Ruth, wrote it on the chalkboard. Hence Tamara took the paper where the problem was written and read it. However, she did it in a particular way: She put herself in the problem instead of John. Tamara was reading the problem in first person, so she was trying to solve the problem as if this was a real situation for her. Ruth and Elena implicitly accepted that strategy and they played along. The fictitious situation (John buying and selling a watch) became a role-playing situation were the characters were Tamara, Ruth, Elena, and Victoria. Drawing on our experience working with adults (Díez-Palomar, 2004Díez-Palomar, J., Giménez Rodríguez, J., & García Wehrle, P., 2006) it is plausible to affirm that this strategy is quite common when adult learners try to understand a mathematical concept or process. They put themselves in the situation as a way to interiorize the process of argumentation and create meaning as a consequence of that. Again, language is quite informal: *rojo* (red), *malo* (bad), *bueno* (good) (as in “pero
tienes veinte malos, te quedan cien buenos”; (but you have 20 bad, you have 100 good left). Ruth uses this informal language as a tool to transfer the mathematical meaning, the formal one, to Tamara, but in their “own” words. Dialogue between peers became a crucial aspect in the whole process. Without dialogue (that is, without that space for dialogue which was the classroom), Ruth and Elena would have not been able to build on Tamara’s ZPD. Consequently dialogue (or spaces of dialogue) was a condition sine qua non for adult learning in this example. Finally, Tamara gives an indication that she is starting to understand the problem better, but not quite yet:

Tamara: Si porque aquí perdí veinte, o sea, los veinte que había ganado, se anula… y lo vendí en 120.
[Yes, because here you lost 20, that is, the twenty that you had profited, it is annulled…. And I sold it at 120.]

Tamara: ¡Ay! Sabe.... No carburo.
[Who knows... I cannot do it.]

Vignette 6. Mathematics and emotions

Norberto is from Northern Mexico. He has a college degree in agricultural engineering; thus, his motivation to participate in the workshops is not for him to learn mathematics. Norberto’s nexus with the school is his nephew who at the time of the interview was in 8th grade. The following conversation was part of the first interview with Norberto. The question being discussed is “How was mathematics taught when you went to school?” Norberto had been discussing how strict teachers were, and how physical punishment was one of the most vivid memories he had.

Norberto: Eran, sí eran muy estrictos los maestros. O sería porque, en aquel tiempo, así era el...el régimen de, de los maestros. Y los pasabas al pizarrón y... muchas veces se te olvida algo, pues y, y también el [imita al maestro pegándole al alumno con una vara] el castigo o, el pellizco.
[They were, teachers certainly were strict. Or maybe because, back then, that is how the teacher’s system was. You would (ask them) to come up to the board and... many times, you would forget something, so, (you would also get) the (imitates teacher hitting student with a stick), the punishment, or a pinch.]

José María: ¿Castigo físico?
[Physical punishment?]

Norberto: Físico. Entonces yo creo que ya va creciendo uno como con cierto temor a estar enfrente. Ya vas adquiriendo recelo, ehh, ¿cómo se dice?...psicológicamente pues, ya vas...teniendo miedo al público. Era eso, eso en la primaria ¿no? Pasó nada más, no, no. . .
[Physical. So, I think one (as a result) grows somehow afraid of being in front (of people). You start getting distrustful, ehh, how do you say it?... psychologically, you start getting afraid of the public. That was, in elementary, right? It only happened, no, no. . .

Norberto: Muchas veces sabía, pero ya sentía miedo, y como que se bloqueaba uno. Pues, ya no...ni para atrás ni para adelante de ahí, pues, ya.
Norberto’s comment about being punished for making a mistake when at the board seems to have conditioned him and many other parents, judging from their responses, to feeling apprehensive towards going to the board to present their work. This predisposition manifests itself during the workshops: Parents are reluctant to go to the board to explain their solution or to work on a problem in front of the rest of the group. The sentiment is so strong that during a debriefing at the end of one of the modules, parents petitioned not to be asked by the facilitator to go to the board.

Celia- … a mí no me gustaría que me obligaran a pasar allá. No, tengo que ser yo quien diga: “Ahora sí lo voy a hacer aunque lo que diga allí no está bien”, me tengo que sentir segura que sí que quiero pasar. [... I wouldn’t like to be forced to go up. No, it has to be me who says, “okay now I am going to do it even if what I have isn’t right,” I have to feel confident that I do want to go up there.]
(Debriefing, Math For Parents workshop, Oct. 2007)

The individual presentation by participants was intended by the facilitator as a way to showcase different strategies to solve the problem, to recognize the participants’ contributions, and to provide individuals with opportunities to exercise leadership roles and to position themselves as intellectual resources, and hence, as a learning experience. Instead, the results of asking parents to participate in this way during the workshops seem to have the opposite effect: apprehension and maybe even embarrassment. This finding is consistent with research on the role of affect in adult learning (Evans, 2000). Díez-Palomar (2004) provides evidence to affirm that egalitarian spaces of dialogue are a crucial element to overcome this type of situations. We argue, however, that those egalitarian spaces may actually be rather elusive, as power issues and prior experiences are likely to influence these adults’ current learning experiences.

Conclusion

What have we learned about adults learning mathematics from our work with parents? There are several items that seem to have an influence on this process. Here we present six ideas drawing from our analysis.

1. **The context.** Our findings provide some evidence of the importance of context in the learning process. Adults use their everyday experiences to understand mathematical ideas. Vignettes 1 and 3 are evidence of this claim. This conclusion is consistent with prior research (Knowles, 1968; Lave & Wenger, 1991). In addition, our analysis introduces a new element from the context point of view: adults may choose to put themselves into the problem, as in role-playing, to get a better understanding of the situation. Vignette 5 was an example of this.
Statement 1. Learning is always situated (embedded) in a particular context. Consequently, in working with adults we should try to find ways to connect formal mathematics with situations framed on adults’ everyday experiences.

2. The interaction. Researchers have underscored the importance of interactions in the process of learning (Flecha, 2000; Freire, 1970, 1998; Vygotsky, 1978). Individual learning is a result of a process of interiorization where individuals get new concepts and ideas through interactions with peers. Vygostky proposed the ZPD to explain how this process works. Flecha has provided evidence that suggests intersubjective interaction as a crucial factor for learning. All vignettes provided in this paper are examples of this fact.

Statement 2. Learning is always produced through interactions among learners. Hence in working with adults we should emphasize and promote interactions among the participants.

3. The egalitarian dialogue. Egalitarian dialogue among participants may lead to learning. Researchers have highlighted the importance of dialogue and communication for adult learners (Flecha, 2000; Freire, 1970, 1998; Mezirow, 1997). Dialogue is a way to share different approaches to the knowledge. Teachers and students are both agents in the school practices. As we saw in the vignettes discussed here, learners interact with each other when they are doing mathematics. Egalitarian dialogue problematizes the traditional way to teach (i.e., based on the teacher / facilitator lecturing). The concept of egalitarian dialogue does not mean that every single actor in the classroom has the same knowledge. What it means is that everybody (teachers/facilitators, students (parents)) should have the same possibilities to share their own knowledge, drawing on arguments and not on power relations. In order to achieve this goal it is crucial to establish a place where participants could feel free and safe to share their ideas. Parents’ workshops discussed in this paper were examples of these kinds of places in that we tried to ensure that everybody has a voice. The vignettes given in this paper provide evidence of participants interacting and sharing their ideas. We do not claim, however, that these are examples of egalitarian dialogues, as power issues and affective issues do play a role. For example, in Vignette 2 and 3, we see Marisol dominating the dialogue and Marisol could be seen as being in a position of power, not only because she works at the school but also because she was the one responsible for the recruitment of these parents to these workshops. Vignette 6 illustrates that this place has to be safe for learners, especially for adult learners because many of them may have had negative experiences with school mathematics. We claim that a place where participants are free to share different approaches to learning and to solving a mathematical problem produces more and better learning. Vignette 5 is an example of that finding: Tamara was unable to understand how John gets $40 after selling the watch several times. But Tamara persisted in her wanting to understand. She felt
comfortable enough to say that she did not understand and seemed to be genuinely interested in trying to make sense of the problem. Then Ruth took the lead and tried to explain to Tamara how John was able to earn $40 dollars. Between Tamara and Ruth they used role-playing (supported by comments by Elena and Victoria). This vignette illustrates a team effort at trying to reach a shared understanding of the problem.

Statement 3. Learning is not unidirectional from teacher to students. Learning is a social practice where everybody participates. For this reason, in working with adult learners we should strive to build spaces where adults can feel comfortable and safe to participate and to share their own knowledge.

4. **The prior experience.** Knowles (1968) wrote that the difference between adults and children is that adults have previous experiences that mediate their process of learning. We cannot address a school topic without having this in mind (Plaza et al., 2004). Data discussed here provide evidence that illustrates this conclusion. Luisa’s group graphical representation, Tamara’s interpretation of how people were distributed in the chairs, Marisol’s idea of slope, Ruth’s approach to the watch problem; all of them are particular examples of how adults make sense to mathematical ideas through their everyday knowledge. Some of this knowledge could be understood as funds of knowledge (González et al., 2001), that is: community-based knowledge (Vignette 4). Other examples illustrate individual knowledge accumulated through experience (Vignettes 1, 3, and 5). In any case previous experience becomes an element that has to be considered when working with adults.

Statement 4. Learning is always connected to our prior experience. Consequently, in working with adult learners we need to draw on everyday examples that reflect and build on learners’ prior experiences.

5. **Concrete vs. abstract mathematics.** Plaza et al. (2004) affirm that adult people prefer concrete examples rather than abstract mathematics. This does not mean that we should shy away from abstract mathematics when working with adults. The parents in these vignettes made connections to their everyday experiences and seemed to enjoy visual approaches, such as the dance metaphor in Vignette 2, as a way to make sense of a mathematical concept (the median in that case). But we know from prior research that parents want to learn the “abstract” mathematics too (Benn, 1997; Civil, 1999; 2004). This may be the case for many adults as one of their motivations for learning mathematics may be to be able to help their children with school work as well as for themselves to further their own learning.

Statement 5. We need to challenge this idea of concrete vs. abstract mathematics. In working with adult learners, we should stress sense-making strategies while ensuring that we address their desire of wanting to learn academic mathematics.
6. *Emotions.* The role of affect in adult mathematics education is well documented (Evans, 2000). For many adults who are returning to studying mathematics was not a positive experience as school children. They find it difficult to understand; some people fear it and sometimes it is even invisible (Niss, 1994). Many people feel that they do not use mathematics although they do (Diez-Palomar, 2004). Vignette 6, Norberto’s recollection of his experiences in school is unfortunately rather common for many adult learners, at least in our research context. Experiences such as Norberto’s may lead to a rejection of school practices and affect subsequent experiences as mathematics learners. Consequently, these previous negative experiences may become a barrier that we have to address.

Statement 6. Emotions play a crucial role in learning and adults are no exception. Learning. Accordingly, in working with adult learners we need to address these kinds of situations right upfront, in order to overcome the emotional barrier that mathematics (and even school in general) represents for some adult learners.

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