Exploring the Mathematical Thinking of Bilingual Primary-Grade Students: CGI Problem Solving from Kindergarten Through 2nd Grade

Mary E. Marshall, Ph.D.
University of New Mexico/John Adams Middle School
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Why Latina/o students? Why mathematics?

• Statement of the problem:
  – Large-scale Latina/o underachievement in mathematics is unacceptable (NAEP, 2005).
  – Success in mathematics is critical for academic and career opportunities across disciplines (Gutstein, 2003; Kitchen, 2007; Secada, 1995).
  – Nationally, classrooms are not meeting the needs of Latina/o students (Education Trust, 2003).

• CEMELA: Center for the Mathematics Education of Latinos/as
Equity in Mathematics Education

  – *All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study - and support to learn mathematics.*

• For all students this means relevant, engaging and intellectually challenging activities (Secada, 1987).

• For Latina/o students this includes recognition of their language and culture as learning resources (Secada & De La Cruz, 1996).

Purpose of this Study

• Explore students’ mathematical thinking over time when they have access to equitable teaching practices that honor their language and culture.

• Highlight young students’ unique and powerful ways of making sense of word problems.

• Demonstrate that Latina/o students have great potential for success in mathematics.
Research Questions

– How do bilingual primary-grade students learning mathematics in Spanish language environments communicate their mathematical thinking during CGI problem solving over the course of 3 years?

– What does student communication in the form of the strategies they use to solve the problems, the tools and materials their choose as aids, and students’ verbal explanations reveal about how they are making sense of the mathematics in the CGI problems?

Theoretical Foundation

• Complementary theories:
  – Sociocultural Theory and System of Meaning (Vygotsky, 1987; Mahn, 2009)
    • Conceptual development results from the interplay between culturally defined external and developing internal systems of meaning.
  – Cognitively Guided Instruction [CGI] (Carpenter et al., 1993, 1996, 1999)
    • Contextualized problem solving framework that bridges students’ informal knowledge and experience to formal mathematical concepts.
System of Meaning from Sociocultural Theory (Mahn, 2009; Vygotsky, 1987)

• Internal sense making and conceptual development is dynamically linked to external linguistic and cultural experiences.
• Language mediates between external knowledge and internal understanding. The meaning contained in a word becomes a generalization/concept.
• Thinking and speaking processes are unified around word meanings and result in verbal thinking.
• Formal scientific concepts organize and restructure informal spontaneous concepts in the system.

Cognitively Guided Instruction [CGI]

• Framework for understanding student thinking.
  – Specific problem types defined by actions or relationships between numbers.
  – Generalization of student strategies and trajectory of development from direct modeling to counting, use of facts, and invented approaches.
  – Advanced strategies are progressive abstractions of previous strategies.

• “Our major thesis is that children bring to school informal or intuitive knowledge of mathematics that can serve as the basis for developing much of the formal mathematics of the primary school curriculum” (Carpenter et al., 1996, p. 6)
CGI Problem Type Examples

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Join Result Unknown</td>
<td>(6 + 6 = __)</td>
</tr>
<tr>
<td>Separate Result Unknown</td>
<td>(13 - 5 = __)</td>
</tr>
<tr>
<td>Join Change Unknown</td>
<td>(7+__ = 11)</td>
</tr>
<tr>
<td>Compare &amp; Part-Part Whole</td>
<td></td>
</tr>
<tr>
<td>Multiplication</td>
<td>(6 x 3 = ___)</td>
</tr>
<tr>
<td>Partitive Division</td>
<td>(15 ÷ 3 = ___)</td>
</tr>
<tr>
<td>Measurement Division</td>
<td>(10 ÷ 2 = ___)</td>
</tr>
<tr>
<td>Multi-Step</td>
<td>(2 x 4) - 3 = ___</td>
</tr>
</tbody>
</table>

Methodology

- Qualitative study in grounded theory tradition (Creswell, 1988)
  - Purpose: How do students make meaning of the mathematics during CGI problem solving?
  - Saturation of data collection, extensive interviews
  - Rigorous coding scheme to develop themes
- Data collected from individual student CGI interviews over 3-year period
Research Site and Participants

• La Joya Elementary School
  – Urban setting serving predominantly Mexican immigrant families, 86% Spanish speakers
  – 100% free or reduced breakfast and lunch
• Student participants
  – 8 students followed for 3 years, K - 2nd grade
  – 6 girls, 2 boys with Spanish as their first language
  – Range of ability in mathematics and range of English proficiency
  – All had classroom CGI problem solving in kindergarten and 1st grade

The Participants and the Research Team

Gerardo, Brisa, Gina, Jenna, Ana, Dolores, Yolanda, Omar
Four Participants Analyzed Using Five Problem Types

Gerardo
- Join Change/Start Unknown
- Compare
- Part-Part-Whole
- Multiplication
- Partitive Division

Yolanda

Omar

Coding Categories

- CGI strategy: Direct modeling or counting.
  - “Gradually over a period of time children replace concrete Direct Modeling strategies with more efficient Counting strategies, and the use of Counting strategies is an important marker in the development of number concepts” (Carpenter et al., 1999, p. 28).
- Valid strategy for the problem, tools used
- Correct answer
- Explanation of their thinking/solution
  - Focus on objects or number sequence?
  - Chapter 4: Explains their language choices.
Analysis: Trends Emerge in CGI Strategies

- Direct modeling vs. counting: 5 problem types, 3 years
  - Omar and Yolanda - counting preference
  - Gina and Gerardo - direct modeling preference
- Note: Trends show a continuum, not a dichotomy.

<table>
<thead>
<tr>
<th>Student</th>
<th>Number of Problems</th>
<th>Number Correct</th>
<th>Direct Model and Combinations of DM and Counting</th>
<th>Advanced Strategies: Counting, Recalled Fact, +, -, or Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omar</td>
<td>37</td>
<td>33 (89%)</td>
<td>5 (14%)</td>
<td>32 (86%)</td>
</tr>
<tr>
<td>Yolanda</td>
<td>39</td>
<td>33 (85%)</td>
<td>10 (26%)</td>
<td>29 (74%)</td>
</tr>
<tr>
<td>Gerardo</td>
<td>35</td>
<td>31 (94%)</td>
<td>24 (69%)</td>
<td>11 (31%)</td>
</tr>
<tr>
<td>Gina</td>
<td>36</td>
<td>35 (97%)</td>
<td>30 (83%)</td>
<td>6 (17%)</td>
</tr>
</tbody>
</table>

Trends were apparent in 1st grade.

(Table created in April, 2008)

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Brisa</td>
<td>0%</td>
<td>92%</td>
<td>33%</td>
<td>89%</td>
</tr>
<tr>
<td>Ana</td>
<td>0%</td>
<td>86%</td>
<td>22%</td>
<td>96%</td>
</tr>
<tr>
<td>Yolanda</td>
<td>71%</td>
<td>71%</td>
<td>75%</td>
<td>92%</td>
</tr>
<tr>
<td>Omar</td>
<td>64%</td>
<td>86%</td>
<td>81%</td>
<td>86%</td>
</tr>
<tr>
<td>Jenna</td>
<td>0%</td>
<td>63%</td>
<td>36%</td>
<td>86%</td>
</tr>
<tr>
<td>Gina</td>
<td>36%</td>
<td>93%</td>
<td>43%</td>
<td>100%</td>
</tr>
<tr>
<td>Gerardo</td>
<td>31%</td>
<td>81%</td>
<td>44%</td>
<td>90%</td>
</tr>
<tr>
<td>Averages</td>
<td>29%</td>
<td>82%</td>
<td>48%</td>
<td>91%</td>
</tr>
</tbody>
</table>
Analysis: Trends Continued in All Coding Categories

- Gina and Gerardo
  - Solve with direct models
  - Model even as they incorporated counting strategies
  - Explain in terms of objects
  - Use drawing, cubes or base ten blocks
- Yolanda and Omar
  - Solve with counting strategies
  - Difficulty falling back to direct models
  - Explain in terms of number sequences
  - Use number lines and 100s charts

Findings: Students are thinking differently about the numbers.

- I propose that students are making sense of the numbers in the CGI problems in different ways.
- “There is more than one concept of number; students need to learn to shift between meanings” (Fuson, 1988).
  - **Cardinal** meaning: a **discrete** set of objects
  - **Ordinal** meaning: a position in a **sequence**
- Gina and Gerardo tend to make sense of the numbers from a cardinal perspective so they build models.
- Yolanda and Omar tend to make sense from an ordinal perspective so they use counting and sequencing.
Findings: Tendencies Indicate Sequential versus Discrete Thinking

<table>
<thead>
<tr>
<th>Omar and Yolanda: The Sequential Thinkers</th>
<th>Gina and Gerardo: The Discrete Thinkers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting strategies, mental math, number operations.</td>
<td>Direct modeling. Later combined models with counting strategies.</td>
</tr>
<tr>
<td>Began with a number then worked up or down sequentially, even in compare.</td>
<td>Began with part-part-whole models. Manipulated sets.</td>
</tr>
<tr>
<td>Unsure of direct models, not first choice, many times did not fall back</td>
<td>Confidence in direct models. Models helped them self-correct.</td>
</tr>
<tr>
<td>successfully.</td>
<td></td>
</tr>
<tr>
<td>Confidence in mental math and counting. Remembered numbers mentally.</td>
<td>Needed numbers repeated. Remembered numbers by referring to models.</td>
</tr>
<tr>
<td>Explained in terms of counting sequences. Rarely referred to the story.</td>
<td>Explained in terms of objects in story. Gave sophisticated explanations and reflected on their own thinking.</td>
</tr>
</tbody>
</table>

Findings: Summary

- The way children approach CGI problem solving reflects the way they make sense of the numbers in the problems.

- Children make sense of the numbers in different ways and in turn their problem-solving strategies differ, depending on if they attach a cardinal meaning to the numbers, an ordinal meaning, or are shifting between both meanings (Fuson, 1988).
Video Example 1: All students solve the same Compare Difference Unknown in 1st grade.

- Compare (16, 10)
  - You have 16 toys and your friend has 10 toys. Who has more? How many more?

- Video:
  - Omar just knows the answer.
  - Yolanda counts up on her fingers.
  - Gerardo draws each set and counts the extras.
  - Gina builds a model with cubes and matches sets.

In addition, each child is unique.

- Children bring different strengths to CGI problem solving:
  - Omar decomposes large numbers in his head and uses this to add and subtract.
  - Yolanda connects algorithmic approaches and formal symbolic representation to a variety of problem types. Looks for efficient solutions.
  - Gerardo gives detailed explanations, is flexible.
  - Gina applies part-part-whole models to a variety of problem situations. Puts her own spin on solutions.

- Children show creativity in solutions (Secada & De La Cruz, 1996).
Video Example 2: Individual student strengths continue through 2nd grade.

• Omar and Gerardo solve Join Start Unknown
  – Omar counts up in his head.
  – Gerardo solves and explains in terms of sets.
    • I make a mistake in Spanish and Sandra corrects me.
    • I ask him to use the base ten blocks.

Video Example 2 cont.

• Yolanda and Gina solve Multiplication
  – Yolanda solves with 100s chart and sequencing.
  – Gina draws a direct model and then uses counting.
    • She recognizes Chicken Little in English.
Implications for CGI Theory

• The trajectory outlined in the CGI literature from direct modeling to counting is more complex for these 4 children. Note that:
  – Omar and Yolanda cannot always fall back successfully to direct modeling.
  – Gina and Gerardo do not leave behind direct modeling as they incorporate counting.
• Whether children can use both direct models and counting strategies on the same problem provides valuable assessment information.
  – Can students shift between cardinal and ordinal meanings?

Implications for Mathematics Education

• Children think in more than one way about numbers (Fuson, 1988).
• Children need experiences in multiple approaches and problem types.
• Teachers need to know that children’s conceptions about the numbers influence their problem solving.
• Children should be able to flexibly shift between sequential and discrete thinking.
Implications for Sociocultural Theory

• We can peer into children’s system of meaning about mathematics word problems during CGI problem solving.
• How students approach the problems and how they explain their thinking reveals how they are making sense of the numbers in the problems (Vygotsky, 1987; Mahn, 2009).

Implications for Native Language Instruction

• From my final chapter, p. 212:
  – The words that mediated their thoughts became part of the external expression of their thinking. There was no conflict between the language they used for learning mathematics and the language they used for thinking. The four students in this study had the power of two languages to make sense of the mathematics in the CGI problems.

  • (Cummins, 2000; Khisty, 1997; Mahn, 2009; Moschkovich, 2002; Vygotsky, 1987)
Implications for Equity in Mathematics Education

• Powerful thinking emerges from equitable learning environments.
• There is great potential in all children.

Areas for Further Research

• How can CGI problem solving facilitate the shift between sequential and discrete thinking?
• When children can shift between cardinal and ordinal meanings how is their overall mathematical development affected?
• I need to further explore how children develop whole number concepts.
  – What does the mathematics education literature say?
The Final Word

• Gerardo talks about feeling successful.