In Exercises 28–29, evaluate the definite integral exactly [as in $\ln(3\pi)$], using the Fundamental Theorem, and numerically [$\ln(3\pi) \approx 2.243$]:

28. $\displaystyle\int_{-3}^{-1} \frac{2}{r^3}\, dr$

29. $\displaystyle\int_{-\pi/2}^{\pi/2} 2\cos\phi\, d\phi$

For Exercises 30–35, find an antiderivative $F(x)$ with $F'(x) = f(x)$ and $F(0) = 4$.

30. $f(x) = x^2$

31. $f(x) = x^3 + 6x^2 - 4$

32. $f(x) = \sqrt{x}$

33. $f(x) = e^x$

34. $f(x) = \sin x$

35. $f(x) = \cos x$

36. Use the fact that $(x^x)' = x^x(1 + \ln x)$ to evaluate exactly: $\displaystyle\int_1^3 x^x(1 + \ln x)\, dx$.

37. Show that $y = x + \sin x - \pi$ satisfies the initial value problem

$$\frac{dy}{dx} = 1 + \cos x, \quad y(\pi) = 0.$$

38. Show that $y = x^n + A$ is a solution of the differential equation $y' = nx^{n-1}$ for any value of A.

In Exercises 39–42, find the general solution of the differential equation.

39. $\dfrac{dy}{dx} = x^3 + 5$

40. $\dfrac{dy}{dx} = 8x + \dfrac{1}{x}$

41. $\dfrac{dW}{dt} = 4\sqrt{t}$

42. $\dfrac{dr}{dp} = 3\sin p$

In Exercises 43–46, find the solution of the initial value problem.

43. $\dfrac{dy}{dx} = 6x^2 + 4x, \quad y(2) = 10$

44. $\dfrac{dP}{dt} = 10e^t, \quad P(0) = 25$

45. $\dfrac{ds}{dt} = -32t + 100, \quad s = 50$ when $t = 0$

46. $\dfrac{dq}{dz} = 2 + \sin z, \quad q = 5$ when $z = 0$

Find the derivatives in Exercises 47–48.

47. $\dfrac{d}{dt} \displaystyle\int_t^\pi \cos(z^3)\, dz$

48. $\dfrac{d}{dx} \displaystyle\int_x^1 \ln t\, dt$

Problems

49. Use Figure 6.35 and the fact that $F(2) = 3$ to sketch the graph of $F(x)$. Label the values of at least four points.
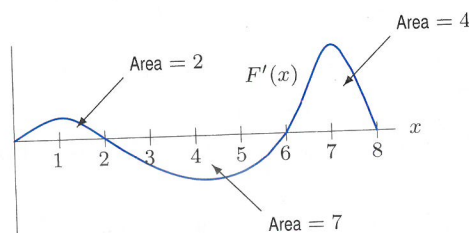

Figure 6.35

50. The vertical velocity of a cork bobbing up and down on the waves in the sea is given by Figure 6.36. Upward is considered positive. Describe the motion of the cork at each of the labeled points. At which point(s), if any, is the acceleration zero? Sketch a graph of the height of the cork above the sea floor as a function of time.
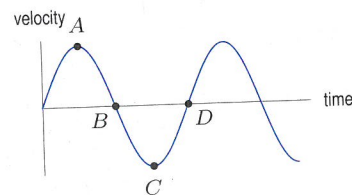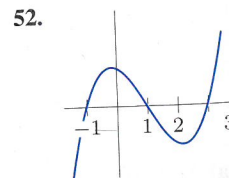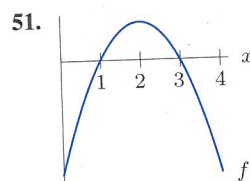

Figure 6.36

In Problems 51–52, a graph of f is given. Let $F'(x) =$.

(a) What are the x-coordinates of the critical points of $F(x)$?

(b) Which critical points are local maxima, which are minima, and which are neither?

(c) Sketch a possible graph of $F(x)$.

51.

52.

CHAPTER SUMMARY (see also Ready Reference at the end of the book)

- **Constructing antiderivatives**
  Graphically, numerically, analytically.
- **The family of antiderivatives**
  The indefinite integral.
- **Differential equations**

Initial value problems, uniform motion.

- **Construction theorem (Second Fundamental Theorem of Calculus)**
  Constructing antiderivatives using definite integrals.
- **Equations of motion**

REVIEW EXERCISES AND PROBLEMS FOR CHAPTER SIX

Exercises

1. The graph of a derivative $f'(x)$ is shown in Figure 6.32.
   Fill in the table of values for $f(x)$ given that $f(0) = 2$.

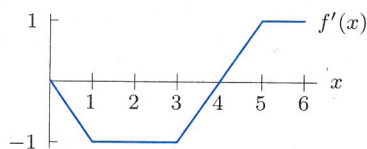| x	0	1	2	3	4	5	6
$f(x)$	2						

Figure 6.32: Graph of f', not f

Figure 6.34

2. Figure 6.33 shows f. If $F' = f$ and $F(0) = 0$, find $F(b)$ for $b = 1, 2, 3, 4, 5, 6$.
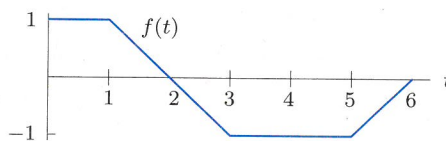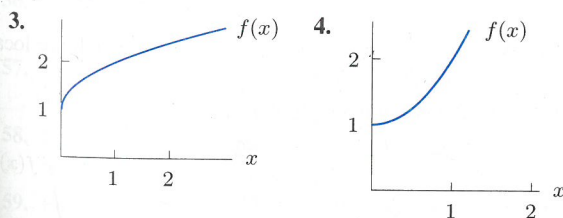
Figure 6.33

In Exercises 3–4, graph $F(x)$ such that $F'(x) = f(x)$ and $F(0) = 0$.

3.

4.

5. (a) Using Figure 6.34, estimate $\int_0^7 f(x)\,dx$.
   (b) If F is an antiderivative of the same function f and $F(0) = 25$, estimate $F(7)$.
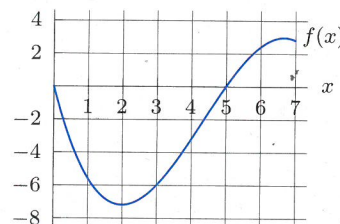
In Exercises 6–27, find the indefinite integrals.

6. $\displaystyle\int 5x\,dx$

7. $\displaystyle\int x^3\,dx$

8. $\displaystyle\int \sin\theta\,d\theta$

9. $\displaystyle\int (x^3 - 2)\,dx$

10. $\displaystyle\int \left(t^2 + \frac{1}{t^2}\right)dt$

11. $\displaystyle\int \frac{4}{t^2}\,dt$

12. $\displaystyle\int (x^2 + 5x + 8)\,dx$

13. $\displaystyle\int 4\sqrt{w}\,dw$

14. $\displaystyle\int (4t + 7)\,dt$

15. $\displaystyle\int \cos\theta\,d\theta$

16. $\displaystyle\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}}\right)dt$

17. $\displaystyle\int \left(x + \frac{1}{\sqrt{x}}\right)dx$

18. $\displaystyle\int (\pi + x^{11})\,dx$

19. $\displaystyle\int \left(3\cos t + 3\sqrt{t}\right)dt$

20. $\displaystyle\int \left(\frac{y^2 - 1}{y}\right)^2 dy$

21. $\displaystyle\int \frac{1}{\cos^2 x}\,dx$

22. $\displaystyle\int \left(\frac{2}{x} + \pi\sin x\right)dx$

23. $\displaystyle\int \left(\frac{x^2 + x + 1}{x}\right)dx$

24. $\displaystyle\int 5e^z\,dz$

25. $\displaystyle\int 2^x\,dx$

26. $\displaystyle\int (3\cos x - 7\sin x)\,dx$

27. $\displaystyle\int (2e^x - 8\cos x)\,dx$