71. $\int \frac{1}{\sin^3(2x)}\,dx$

72. $\int \frac{dr}{r^2-100}$

73. $\int y^2\sin(cy)\,dy$

74. $\int e^{-ct}\sin kt\,dt$

75. $\int e^{5x}\cos(3x)\,dx$

76. $\int \left(x^{\sqrt{k}}+\sqrt{k}^{x}\right)\,dx$

77. $\int \sqrt{3+12x^2}\,dx.$

78. $\int \frac{1}{\sqrt{x^2-3x+2}}\,dx$

79. $\int \frac{x^3}{x^2+3x+2}\,dx$

80. $\int \frac{x^2+1}{x^2-3x+2}\,dx$

81. $\int \frac{dx}{ax^2+bx}$

82. $\int \frac{ax+b}{ax^2+2bx+c}\,dx$

83. $\int \left(\frac{x}{3}+\frac{3}{x}\right)^2\,dx$

84. $\int \frac{2^t}{2^t+1}\,dt$

85. $\int 10^{1-x}\,dx$

86. $\int (x^2+5)^3\,dx$

87. $\int v\arcsin v\,dv$

88. $\int \sqrt{4-x^2}\,dx$

89. $\int \frac{z^3}{z-5}\,dz$

90. $\int \frac{\sin w\cos w}{1+\cos^2 w}\,dw$

91. $\int \frac{1}{\tan(3\theta)}\,d\theta$

92. $\int \frac{x}{\cos^2 x}dx$

93. $\int \frac{x+1}{\sqrt{x}}\,dx$

94. $\int \frac{x}{\sqrt{x+1}}\,dx$

95. $\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{x}}\,dx$

96. $\int \frac{e^{2y}}{e^{2y}+1}\,dy$

97. $\int \frac{z}{(z^2-5)^3}\,dz$

98. $\int \frac{z}{(z-5)^3}dz$

99. $\int \frac{(1+\tan x)^3}{\cos^2 x}\,dx$

100. $\int \frac{(2x-1)e^{x^2}}{e^x}dx$

101. $\int (2x+1)e^{x^2}e^x\,dx$

102. $\int \sqrt{y^2-2y+1}(y-1)\,dy$

103. $\int \sin x(\sqrt{2+3\cos x})dx$

104. $\int (x^2-3x+2)e^{-4x}\,dx$

105. $\int \sin^2(2\theta)\cos^3(2\theta)\,d\theta$

106. $\int \cos(2\sin x)\cos x\,dx$

107. $\int (x+\sin x)^3(1+\cos x)\,dx$

108. $\int \left(2x^3+3x+4\right)\cos(2x)\,dx$

109. $\int \sinh^2 x\cosh x\,dx$

110. $\int (x+1)\sinh(x^2+2x)\,dx$

For Exercises 111–124, evaluate the definite integrals using the Fundamental Theorem of Calculus and check your answers numerically.

111. $\int_0^1 x(1+x^2)^{20}\,dx$

112. $\int_4^1 x\sqrt{x^2+4}\,dx$

113. $\int_0^{\pi} \sin\theta(\cos\theta+5)^7\,d\theta$

114. $\int_0^1 \frac{x}{1+5x^2}\,dx$

115. $\int_1^2 \frac{x^2+1}{x}\,dx$

116. $\int_1^3 \ln(x^3)\,dx$

117. $\int_1^e (\ln x)^2\,dx$

118. $\int_{-\pi}^{\pi} e^{2x}\sin 2x\,dx$

119. $\int_0^{10} ze^{-z}\,dz$

120. $\int_{-\pi/3}^{\pi/4} \sin^3\theta\cos\theta\,d\theta$

121. $\int_1^8 \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx$

122. $\int_0^1 \frac{dx}{x^2+1}$

123. $\int_{-\pi/4}^{\pi/4} \cos^2\theta\sin^5\theta\,d\theta$

124. $\int_{-2}^{0} \frac{2x+4}{x^2+4x+5}\,dx$

125. Use partial fractions on $\frac{1}{x^2-1}$ to find $\int \frac{1}{x^2-1}dx.$

126. **(a)** Use partial fractions to find $\int \frac{1}{x^2-x}dx.$

(b) Show that your answer to part (a) agrees with the answer you get by using the integral tables.

127. Use partial fractions to find $\int \frac{1}{x(L-x)}\,dx$, where L is constant.

Evaluate the integrals in Exercises 128–139 using partial fractions or a trigonometric substitution (a and b are positive constants).

128. $\int \frac{1}{(x-2)(x+2)}dx$

129. $\int \frac{1}{\sqrt{25-x^2}}dx,$

130. $\int \frac{1}{x(x+5)}dx$

131. $\int \frac{1}{\sqrt{1-9x^2}}dx$

132. $\displaystyle\int \frac{2x+3}{x(x+2)(x-1)}\,dx$ **133.** $\displaystyle\int \frac{3x+1}{x(x^2-1)}\,dx$

134. $\displaystyle\int \frac{1+x^2}{x(1+x)^2}\,dx$ **135.** $\displaystyle\int \frac{1}{x^2+2x+2}\,dx$

136. $\displaystyle\int \frac{1}{x^2+4x+5}\,dx$ **137.** $\displaystyle\int \frac{1}{\sqrt{a^2-(bx)^2}}\,dx$

138. $\displaystyle\int \frac{\cos x}{\sin^3 x+\sin x}\,dx$ **139.** $\displaystyle\int \frac{e^x}{e^{2x}-1}\,dx$

Calculate the integrals in Exercises 140–143, if they converge. You may calculate the limits by appealing to the dominance of one function over another, or by l'Hopital's rule.

140. $\displaystyle\int_0^4 \frac{dx}{\sqrt{16-x^2}}$ **141.** $\displaystyle\int_0^3 \frac{5}{x^2}\,dx$

142. $\displaystyle\int_0^2 \frac{1}{x-2}\,dx$ **143.** $\displaystyle\int_0^8 \frac{1}{\sqrt[3]{8-x}}\,dx$

For Exercises 144–157 decide if the integral converges or diverges. If the integral converges, find its value or give a bound on its value.

144. $\displaystyle\int_4^\infty \frac{dt}{t^{3/2}}$ **145.** $\displaystyle\int_{10}^\infty \frac{dx}{x\ln x}$

146. $\displaystyle\int_0^\infty we^{-w}\,dw$ **147.** $\displaystyle\int_{-1}^1 \frac{1}{x^4}\,dx$

148. $\displaystyle\int_{-\pi/4}^{\pi/4} \tan\theta\,d\theta$ **149.** $\displaystyle\int_2^\infty \frac{1}{4+z^2}\,dz$

150. $\displaystyle\int_{10}^\infty \frac{1}{z^2-4}\,dz$ **151.** $\displaystyle\int_{-5}^{10} \frac{dt}{\sqrt{t+5}}$

152. $\displaystyle\int_0^{\pi/2} \frac{1}{\sin\phi}\,d\phi$ **153.** $\displaystyle\int_0^{\pi/4} \tan 2\theta\,d\theta$

154. $\displaystyle\int_1^\infty \frac{x}{x+1}\,dx$ **155.** $\displaystyle\int_0^\infty \frac{\sin^2\theta}{\theta^2+1}\,d\theta$

156. $\displaystyle\int_0^\pi \tan^2\theta\,d\theta$ **157.** $\displaystyle\int_0^1 (\sin x)^{-3/2}\,dx$

Problems

In Problems 158–160, find the exact area.

158. Under $y=(e^x)^2$ for $0\le x\le 1$.

159. Between $y=(e^x)^3$ and $y=(e^x)^2$ for $0\le x\le 3$.

160. Between $y=e^x$ and $y=5e^{-x}$ and the y-axis.

161. The curves $y=\sin x$ and $y=\cos x$ cross each other infinitely often. What is the area of the region bounded by these two curves between two consecutive crossings?

162. Evaluate $\int_0^2 \sqrt{4-x^2}\,dx$ using its geometric interpretation.

In Problems 163–164, find a substitution w and constants k, p so that the integral has the form $\int kw^p\,dw$.

163. $\displaystyle\int 3x^4\sqrt{3x^5+2}\,dx$ **164.** $\displaystyle\int \frac{5\sin(3\theta)\,d\theta}{\cos^3(3\theta)}$

In Problems 165–168, give the substitution and the values of any constants to rewrite the integral in the desired form.

165. $\displaystyle\int \frac{dx}{(2x-3)(3x-2)}$ as $\displaystyle\int \left(\frac{A}{2x-3}+\frac{B}{3x-2}\right)dx$

166. $\displaystyle\int (x^2+x)\cos(0.5x-1)\,dx$ as $\displaystyle\int p(u)\cos(u)\,du$, where $p(u)$ is a polynomial

167. $\displaystyle\int x^3e^{-x^2}\,dx$ as $\displaystyle\int kue^u\,du$

168. $\displaystyle\int \frac{\cos^4\left(\sqrt{x}\right)\sin\sqrt{x}\,dx}{\sqrt{x}}$ as $\displaystyle\int ku^n\,du$

In Problems 169–172, explain why the following pairs of indefinite integrals are really, despite their apparent dissimilarity, different expressions of the same problem. You do not need to evaluate the integrals.

169. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ and $\displaystyle\int \frac{x\,dx}{\sqrt{1-x^4}}$

170. $\displaystyle\int \frac{dx}{x^2+4x+4}$ and $\displaystyle\int \frac{x}{(x^2+1)^2}\,dx$

171. $\displaystyle\int \frac{x}{1-x^2}\,dx$ and $\displaystyle\int \frac{1}{x\ln x}\,dx$

172. $\displaystyle\int \frac{x}{x+1}\,dx$ and $\displaystyle\int \frac{1}{x+1}\,dx$

In Problems 173–174, show the two integrals are equal using a substitution.

173. $\displaystyle\int_0^2 e^{-w^2}\,dw = \int_0^1 2e^{-4x^2}\,dx$

174. $\displaystyle\int_0^3 \frac{\sin t}{t}\,dt = \int_0^1 \frac{\sin 3t}{t}\,dt$

175. A function is defined by $f(t)=t^2$ for $0\le t$ $f(t)=2-t$ for $1<t\le 2$. Compute $\int_0^2 f(t)$

176. **(a)** Find $\int (x+5)^2\,dx$ in two ways:
(i) By multiplying out
(ii) By substituting $w = x+5$
(b) Are the results the same? Explain.

177. Suppose $\int_{-1}^{1} h(z)\,dz = 7$, and that $h(z)$ is even. Calculate the following:

(a) $\int_0^1 h(z)\,dz$ **(b)** $\int_{-4}^{-2} 5h(z+3)\,dz$

178. Find the average (vertical) height of the shaded area in Figure 7.26.

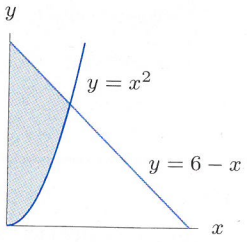

Figure 7.26

179. Find the average (horizontal) width of the shaded area in Figure 7.26.

180. **(a)** Find the average value of the following functions over one cycle:
(i) $f(t) = \cos t$
(ii) $g(t) = |\cos t|$
(iii) $k(t) = (\cos t)^2$
(b) Write the averages you have just found in ascending order. Using words and graphs, explain why the averages come out in the order they do.

181. What, if anything, is wrong with the following calculation?

$$\int_{-2}^{2} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{-2}^{2} = -\frac{1}{2} - \left(-\frac{1}{-2}\right) = -1.$$

182. Let

$$E(x) = \int \frac{e^x}{e^x + e^{-x}}\,dx \text{ and } F(x) = \int \frac{e^{-x}}{e^x + e^{-x}}\,dx.$$

(a) Calculate $E(x) + F(x)$.
(b) Calculate $E(x) - F(x)$.
(c) Use your results from parts (a) and (b) to calculate $E(x)$ and $F(x)$.

183. Using Figure 7.27, put the following approximations to the integral $\int_a^b f(x)\,dx$ and its exact value in order from smallest to largest: LEFT(5), LEFT(10), RIGHT(5), RIGHT(10), MID(10), TRAP(10), Exact value

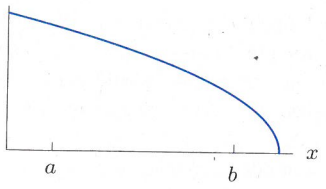

Figure 7.27

184. You estimate $\int_0^{0.5} f(x)dx$ by the trapezoid and midpoint rules with 100 steps. Which of the two estimates is an overestimate, and which is an underestimate, of the true value of the integral if

(a) $f(x) = 1 + e^{-x}$ **(b)** $f(x) = e^{-x^2}$
(c) $f(x)$ is a line

185. **(a)** Using the left rectangle rule, a computer takes two seconds to compute a particular definite integral accurate to 4 digits to the right of the decimal point. How long (in years) does it take to get 8 digits correct using the left rectangle rule? How about 12 digits? 20 digits?
(b) Repeat part (a) but this time assume that the trapezoidal rule is being used throughout.

186. Given that $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$, find $\int_0^\infty x^2 e^{-x^2}\,dx$.

187. A population, P, is said to be growing logistically if the time, T, taken for it to increase from P_1 to P_2 is given by

$$T = \int_{P_1}^{P_2} \frac{k\,dP}{P(L-P)},$$

where k and L are positive constants and $P_1 < P_2 < L$.

(a) Calculate the time taken for the population to grow from $P_1 = L/4$ to $P_2 = L/2$.
(b) What happens to T as $P_2 \to L$?

188. In 2005, the average per-capita income in the US was \$34,586 and increasing at a rate of $r(t) = 1556.37e^{0.045t}$ dollars per year, where t is the number of years since 2005.

(a) Estimate the average per-capita income in 2015.
(b) Find a formula for the average per-capita income as a function of time after 2005.

189. A patient is given an injection of Imitrex, a migraine medicine, at a rate of $r(t) = 2te^{-2t}$ ml/sec, where t is the number of seconds since the injection started.

(a) By letting $t \to \infty$, estimate the total quantity of Imitrex injected.
(b) What fraction of this dose has the patient received at the end of 5 seconds?

REVIEW EXERCISES AND PROBLEMS FOR CHAPTER SEVEN

Exercises

For Exercises 1–4, find an antiderivative.

1. $q(t) = (t+1)^2$
2. $p(\theta) = 2\sin(2\theta)$
3. $f(x) = 5^x$
4. $r(t) = e^t + 5e^{5t}$

For Exercises 5–110, evaluate the following integrals. Assume a, b, c, and k are constants. Exercises 7– 69 can be done without an integral table, as can some of the later problems.

5. $\int (3w+7)\,dw$
6. $\int e^{2r}\,dr$
7. $\int \sin t\,dt$
8. $\int \cos 2t\,dt$
9. $\int e^{5z}\,dz$
10. $\int \cos(x+1)\,dx$
11. $\int \sin 2\theta\,d\theta$
12. $\int (x^3-1)^4 x^2\,dx$
13. $\int \left(x^{3/2} + x^{2/3}\right)\,dx$
14. $\int (e^x + 3^x)\,dx$
15. $\int \frac{1}{e^z}\,dz$
16. $\int \left(\frac{4}{x^2} - \frac{3}{x^3}\right)\,dx$
17. $\int \frac{x^3+x+1}{x^2}\,dx$
18. $\int \frac{(1+\ln x)^2}{x}\,dx$
19. $\int te^{t^2}\,dt$
20. $\int x\cos x\,dx$
21. $\int x^2 e^{2x}\,dx$
22. $\int x\sqrt{1-x}\,dx$
23. $\int y\ln y\,dy$
24. $\int y\sin y\,dy$
25. $\int (\ln x)^2\,dx$
26. $\int e^{0.5-0.3t}\,dt$
27. $\int \sin^2\theta\cos\theta\,d\theta$
28. $\int x\sqrt{4-x^2}\,dx$
29. $\int \frac{(u+1)^3}{u^2}\,du$
30. $\int \frac{\cos\sqrt{y}}{\sqrt{y}}\,dy$
31. $\int \frac{1}{\cos^2 z}\,dz$
32. $\int \cos^2\theta\,d\theta$
33. $\int t^{10}(t-10)\,dt$
34. $\int \tan(2x-6)\,dx$
35. $\int \frac{(\ln x)^2}{x}\,dx$
36. $\int \frac{(t+2)^2}{t^3}\,dt$
37. $\int \left(x^2 + 2x + \frac{1}{x}\right)\,dx$
38. $\int \frac{t+1}{t^2}\,dt$
39. $\int te^{t^2+1}\,dt$
40. $\int \tan\theta\,d\theta$
41. $\int \sin(5\theta)\cos(5\theta)\,d\theta$
42. $\int \frac{x}{x^2+1}\,dx$
43. $\int \frac{dz}{1+z^2}$
44. $\int \frac{dz}{1+4z^2}$
45. $\int \cos^3 2\theta\sin 2\theta\,d\theta$
46. $\int \sin 5\theta\cos^3 5\theta\,d\theta$
47. $\int \sin^3 z\cos^3 z\,dz$
48. $\int t(t-10)^{10}\,dt$
49. $\int \cos\theta\sqrt{1+\sin\theta}\,d\theta$
50. $\int xe^x\,dx$
51. $\int t^3 e^t\,dt$
52. $\int_1^3 x(x^2+1)^{70}\,dx$
53. $\int (3z+5)^3\,dz$
54. $\int \frac{du}{9+u^2}$
55. $\int \frac{\cos w}{1+\sin^2 w}\,dw$
56. $\int \frac{1}{x}\tan(\ln x)\,dx$
57. $\int \frac{1}{x}\sin(\ln x)\,dx$
58. $\int \frac{w\,dw}{\sqrt{16-w^2}}$
59. $\int \frac{e^{2y}+1}{e^{2y}}\,dy$
60. $\int \frac{\sin w\,dw}{\sqrt{1-\cos w}}$
61. $\int \frac{dx}{x\ln x}$
62. $\int \frac{du}{3u+8}$
63. $\int \frac{x\cos\sqrt{x^2+1}}{\sqrt{x^2+1}}\,dx$
64. $\int \frac{t^3}{\sqrt{1+t^2}}\,dt$
65. $\int ue^{ku}\,du$
66. $\int (w+5)^4 w\,dw$
67. $\int e^{\sqrt{2x+3}}\,dx$
68. $\int (e^x+x)^2\,dx$
69. $\int u^2\ln u\,du$
70. $\int \frac{5x+6}{x^2+4}\,dx$