Experimental verification of rank one chaos in switch-controlled smooth Chua's circuit

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Abstract

In this paper, we provide the first experimental proof for the existence of rank one chaos in the switch-controlled smooth Chua's circuit by following a step-by-step procedure given by the theory of rank one chaos. At the center of this procedure is a periodically kicked Hopf limit cycle obtained from the unforced system. The periodic kicking is achieved by adding externally controlled switches to the original smooth Chua's circuit. Experimental results are found to be in perfect agreement with the conclusions of the theory.

Key words: Rank on chaos, switch-controlled, Chua's, nonlinear, Hopf bifurcation.

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1 Introduction

In a sequence of papers published recently, Oksasoglu and Wang have proposed a generic scheme of creating rank one chaos in practical circuits by using periodically controlled switches [1–4]. This chaos scheme is based on a new chaos theory, namely the theory of rank one maps, developed in recent years by Wang and Young [5–7]. The theory of rank one maps is based on Jakobson's theory on quadratic maps [8] and the studies of Benedicks and Carleson on strongly dissipative Hénon maps [9]. The theory of rank one maps has previously been applied to rigorously verify the existence of rank one attractors, and extensive numerical simulations have been conducted in search of the strange attractors implicated under the guidance of the theory [1–4]. The results of these numerical simulations were found to be in perfect match with the conclusions of the theory.

In this paper, we provide the *first experimental evidence* of rank one chaos in a switch-controlled circuit, namely the switch-controlled smooth Chua's circuit [11] as proposed by Oksasoglu and Wang in [4]. Following the procedure outlined in [12], a Chua's circuit with a nonlinear resistor of cubic $v - i$ characteristics is first constructed. The use of a nonlinear resistor with a cubic nonlinearity is necessitated by the fact that a local nonlinearity is needed to create Hopf bifurcations. The parameters of the circuit are so chosen that it has a weakly stable oscillation freshly coming out of a supercritical Hopf bifurcation. Then, switches, controlled by periodic pulses, are added in such a way to modulate the state variables in the circuit. The addition of periodically controlled switches to an existing nonlinear system provides a natural setting for the application of the theory of rank one maps. In other words, the use of periodically controlled switches generates the kicking effect proposed by Wang and Young [7,10] to create rank one chaos. In the range of parameters where the theory of rank one maps applies according to the previous computations in [4], strange attractors do appear experimentally in the way that was predicted by the theory.

A great majority of the existing studies on chaotic attractors are based on breaking the homoclinic loop by small perturbations to yield transverse homoclinic orbit (transversal intersections of the stable and unstable manifolds) in the phase space (see, e.g., [13]). The rank one attractors presented in this paper are, however, of a different kind. They are generated by small disturbances that are periodically applied to a weakly stable limit cycle. When the periodic kicks are introduced (by use of externally controlled switches in this case), the shape of the weakly stable limit cycle is slightly deformed. Then the natural force of shearing created by the nonlinearity of the original system goes to work to exaggerate the initial deformation to create chaos. The dynamical properties of the rank one attractors created that way are dominated by the so-called SRB [14] measures representing the statistical law of the system.

It is also worth noting that the study of the strange attractors in this paper is backed up by a comprehensive theory of dynamics with a long history. The theory itself was little known outside the pure mathematical side of the dynamical systems community, and has only been recently developed into a form that is applicable to concrete systems of differential equations. We refer the reader to a recent tutorial paper [15] for more background information on the theory and its potential applications to circuits and systems.

2 Theoretical setting and implementation approach

In this section, we briefly discuss the setting of the theory and a practical approach introduced in [4] to generically satisfy the requirements of the theory. We first start with an autonomous system given by

$$\frac{d\mathbf{u}}{dt} = f_\mu(\mathbf{u}) \tag{2.1}$$

where $\mathbf{u} \in \mathbb{R}^n, n \geq 2$ represents the system state variables, and $\mu \in \mathbb{R}^m$ the system parameters. It is assumed that there is a $\mu = \mu_0$ at which the system of Eq. (2.1) goes through a supercritical Hopf bifurcation. This system is then modified to obtain the following nonautonomous system:

$$\frac{d\mathbf{u}}{dt} = f_\mu(\mathbf{u}) + \varepsilon\Phi(\mathbf{u})P_{T,p}(t) \tag{2.2}$$

where $P_{T,p}(t)$ is a periodic pulsetrain with a pulsewidth of p and a period of T, $\Phi(\mathbf{u})$ is a function that determines the shape of the forcing, and ε is used to control the magnitude of the forcing. Let $T >> p$ so that a pulse of pulsewidth p is followed by a long relaxation period $T - p$. We regard the system of Eq. (2.2) as the kicked version of the system of Eq. (2.1). When the system of Eq. (2.1) is an electrical system whose state variables are the capacitor voltages and inductor currents, it can be implemented by modulating the state variables through switches externally controlled by $P_{T,p}(t)$. This scheme, depicted in Fig. 1, was proposed by Oksasoglu and Wang in [4]. In Fig. 1, each switch is controlled by the periodic pulsetrain $P_{T,p}(t)$. In this case, the governing equations for the capacitor voltage and the inductor current are given by

$$
\begin{aligned}
C\frac{dv_c}{dt} &= i_s(t) - v_c G_1 P_{T,p}(t) \\
L\frac{di_L}{dt} &= v_s(t) - i_L R_2 P_{T,p}(t)
\end{aligned}
\tag{2.3}
$$

In the scheme of Fig. 1, the resulting $\Phi(\mathbf{u})$, the shape of the forcing, becomes $\Phi(\mathbf{u}) = -\mathbf{u}$.

3 Switch-controlled smooth Chua's circuit

For the experimental investigations of this paper, we apply the above-outlined scheme to the well-known Chua's circuit [11]. The modified circuit, which will, from this point on, be referred to as the switch-controlled smooth Chua's circuit, is depicted in Fig. 2. The switches $\mathbf{S}_i$ are controlled by a periodic pulsetrain with p_0 and T_0 being the pulsewidth and the period, respectively.

Due to the need for a local nonlinearity for Hopf bifurcations to occur, the piecewise linear characteristic of the nonlinear resistor in the original Chua's circuit is replaced with a cubic polynomial one in the switch-controlled circuit. More explicitly, the $v - i$ characteristic of the nonlinear resistor in Fig. 2 is given by

$$
i_n(v_1) = g(v_1) = a_1 v_1 + a_3 v_1^3 \tag{3.1}
$$

The actual realization of this cubic nonlinearity is achieved using the design approach given in [12]. The governing equations for the switch-controlled circuit can be given by

$$\begin{aligned} C_1\frac{dv_1}{dt} =& G(v_2 - v_1) - g(v_1) - G_1 v_1 \\ C_2\frac{dv_2}{dt} =& i + G(v_1 - v_2) - G_2 v_2 \\ L\frac{di}{dt} =& - v_2 - R_3 i \end{aligned} \tag{3.2}$$

for $nT_0 \leq t < nT_0 + p_0$, and by

$$\begin{aligned} C_1\frac{dv_1}{dt} =& G(v_2 - v_1) - g(v_1) \\ C_2\frac{dv_2}{dt} =& i + G(v_1 - v_2) \\ L\frac{di}{dt} =& - v_2 \end{aligned} \tag{3.3}$$

for $nT_0 + p_0 \leq t < (n+1)T_0$, $n = 0, 1, 2, \cdots$. Putting Eqs. (3.2) and (3.3) together, we obtain

$$\begin{aligned} C_1\frac{dv_1}{dt} =& G(v_2 - v_1) - g(v_1) - G_1 v_1 \sum_{n=0}^{\infty} F_{n,p_0,T_0}(t) \\ C_2\frac{dv_2}{dt} =& i + G(v_1 - v_2) - G_2 v_2 \sum_{n=0}^{\infty} F_{n,p_0,T_0}(t) \\ L\frac{di}{dt} =& - v_2 - R_3 i \sum_{n=0}^{\infty} F_{n,p_0,T_0}(t) \end{aligned} \tag{3.4}$$

where

$$F_{n,T_0,p_0}(t) = \begin{cases} 1 & nT_0 \leq t < nT_0 + p_0 \\ 0 & elsewhere. \end{cases} \tag{3.5}$$

By setting

$$x = \frac{v_1}{V_0}, \quad y = \frac{v_2}{V_0}, \quad z = \frac{i}{I_0}, \quad t \rightarrow \frac{t}{\omega_n}, \tag{3.6}$$

we obtain the following dimensionless set of equations

$$
\begin{aligned}
\frac{dx}{dt} =& \alpha[y - h(x)] - \varepsilon_1 x P_{T,p}(t) \\
\frac{dy}{dt} =& \gamma[x - y + \rho z] - \varepsilon_2 y P_{T,p}(t) \\
\frac{dz}{dt} =& -\beta y - \varepsilon_3 z P_{T,p}(t)
\end{aligned}
\tag{3.7}
$$

where

$$
\begin{aligned}
P_{T,p}(t) &= \frac{1}{p}\sum_{n=-\infty}^{\infty} F_{n,T,p}(t), \quad h(x) = b_1 x + b_3 x^3; \\
b_1 &= 1 + \frac{a_1}{G}, \quad b_3 = \frac{a_3 V_0^2}{G}; \\
p &= p_0 \omega_n, \quad T = T_0 \omega_n; \\
\alpha &= \frac{G}{C_1 \omega_n}, \quad \gamma = \frac{G}{C_2 \omega_n} = 1.0; \\
\rho &= \frac{R}{R_n}, \quad R_n = \frac{V_0}{I_0}, \quad \beta = \frac{R_n}{L\omega_n}; \\
\varepsilon_1 &= \frac{\alpha R p}{R_1}, \quad \varepsilon_2 = \frac{\gamma R p}{R_2}, \quad \varepsilon_3 = \frac{\beta R_3 p}{R_n}.
\end{aligned}
\tag{3.8}
$$

Although various single- or multi-switch control schemes can be formulated by setting selected ε_i to zero, in our experimental investigations, we only employ $\mathbf{S_1}$ and $\mathbf{S_2}$ by setting $\varepsilon_3 = 0$.

4 Hopf bifurcation and conditions for rank one chaos

In this section, by following the procedure introduced in [4], we seek for the values of parameters for the system of Eq. (3.7) where rank one chaos is likely to occur. First, we consider the autonomous part of the system of Eq. (3.7) and look for the values of parameters for a supercritical Hopf bifurcation at $(x, y, z) = (0, 0, 0)$. For the computations to follow, we regard ρ as the bifurcation parameter. Observe that at

$$\rho_0 = -\frac{\alpha(b_1 - 1)(\alpha b_1 + 1)}{\beta} > 0, \tag{4.1}$$

the eigenvalues of the linear part of Eq. (3.7) are $\pm i\omega$ and $-(\alpha b_1 + 1)$ where

$$\omega^2 = -\alpha^2 b_1(b_1 - 1) > 0. \tag{4.2}$$

Thus, a necessary condition for a Hopf bifurcation to occur is

$$b_1 \in (0, 1). \tag{4.3}$$

According to the standard theory of Hopf bifurcations, Eq. (3.7) has a center manifold, on which the equation for the flow can be transformed into the following normal form:

$$\frac{dz}{dt} = (a(\mu) + \omega(\mu)\sqrt{-1})z + k_1(\mu)z^2\bar{z} + k_2(\mu)z^3\bar{z}^2 + \cdots \tag{4.4}$$

where $k_1(\mu), k_2(\mu)$ are complex numbers. The fact that there is a well-defined computational process to reach the indicated normal form is important to us. Let us write

$$k_1(\mu) = -E(\mu) + F(\mu)\sqrt{-1}. \tag{4.5}$$

$k_1(0)$ is explicitly calculated in [4]. From the computations in [4], we have

$$E(0) = -c_1(1 + 2\alpha b_1 - \alpha), \quad F(0) = -c_1\frac{\omega}{\alpha b_1}(1 + 2\alpha b_1). \tag{4.6}$$

where

$$c_1 = \frac{-3\alpha b_3}{8b_1(1 + 2b_1\alpha + b_1\alpha^2)}. \tag{4.7}$$

Furthermore, in order to have a weakly stable periodic solution coming out of the origin, it is also necessary to have $E(0) > 0$ yielding

$$\frac{-3\alpha b_3}{8b_1(\alpha^2 b_1 + 2\alpha b_1 + 1)}[1 + 2\alpha b_1 - \alpha] < 0. \tag{4.8}$$

Consequently, for a supercritical Hopf limit cycle to occur, we must have

$$\begin{aligned} b_1 &> \frac{\alpha - 1}{2\alpha}, \quad \text{if} \quad b_3 > 0 \\ b_1 &< \frac{\alpha - 1}{2\alpha}, \quad \text{if} \quad b_3 < 0. \end{aligned} \tag{4.9}$$

According to the theory of [10], in order for rank one attractors to exist we should have a relatively large twist number, which is defined as

$$\tau := \left| \frac{F(0)}{E(0)} \right|. \tag{4.10}$$

Therefore, to find rank one attractors we need to adjust the values of parameters in such a way to make

$$\left| \frac{F(0)}{E(0)} \right| = \left| \frac{Im(k_1)}{Re(k_1)} \right| = \left| \frac{\omega(1 + 2\alpha b_1)}{\alpha b_1(1 + 2\alpha b_1 - \alpha)} \right|. \tag{4.11}$$

large.

In summary, the values of parameters are determined using the following guidelines [15]. Let $\alpha, \beta, \gamma, \rho, b_1, b_3$ be the parameters of the autonomous part of Eq. (3.7), and p, $\varepsilon = \varepsilon_1$, T be the parameters of the periodic forcing. We fix the values of all parameters except T as follows:

(i) *Parameter values for Hopf bifurcation:* $b_3 \neq 0$, $\beta > 0$, $\alpha > 1$ are arbitrarily fixed, and ρ is around $\rho_0 = -\frac{\alpha(b_1-1)(\alpha b_1+1)}{\beta}$.

(ii) *Strong shearing:* choose $b_1 \in (0,1)$ sufficiently close to $b_1 = \frac{\alpha-1}{2\alpha}$ either from above or below depending on the sign of b_3 (see Eq. (4.9) for stability criterion).

(iii) *Parameters of forcing:* choose ε relatively small, e.g., $\varepsilon < 1$.

Following the steps outlined above, to have a supercritical Hopf limit cycle, we choose and fix

$$\begin{aligned} &\alpha = 2.0,\ \beta = 2.0,\ \gamma = 1.0,\ b_3 = -1.0,\ b_1 = 0.242 \\ &\rho_0 = 1.124872,\ \varepsilon_i = 0,\ \rho = \rho_0 - 0.005. \end{aligned} \tag{4.12}$$

A Hopf limit cycle numerically obtained for these values is shown in Fig. 3. In this case, the twist constant is roughly

$$\tau := \left|\frac{F(0)}{E(0)}\right| = 108. \tag{4.13}$$

Two rank one attractors found through numerical simulations by kicking the limit cycle of Fig. 3 are shown in Figs. 4 and 5. The attractor of Fig. 4 is obtained by employing $\mathbf{S_1}$ only with $\varepsilon_1 = 0.5$, $p = 0.5$, and $T = 97.0$. The attractor of Fig. 5 is obtained by using two switches $\mathbf{S_1}$ and $\mathbf{S_2}$ with $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.34$, $p = 0.5$, and $T = 87.5$. For Figs. 4 and 5, part (a) (top) is the plot of an orbit of the time-T map on the $x - y$ plane, part (b) (bottom left) is the plot of the x-coordinate of this orbit versus discrete time k, and part (c) (bottom right) is the frequency spectrum of the x-coordinate for the orbit plotted.

5 Circuit implementation and experimental results

The circuit implementation of the cubic $v - i$ characteristic, $i_n = a_1 v_1 + a_3 v_1^3$, of the nonlinear resistor in Fig. 2 can be accomplished by use of analog multipliers. Here, we follow the same design approach given in [12]. The specific analog multipliers used for this purpose are AD633 of Analog Devices. For Op Amps, AD711s are used. The biasing used for all the active elements is ±5V. The resulting implementation of this cubic $v - i$ characteristic is given in Fig. 6.

For the controlled switches of Fig. 2, Texas Instruments' CD4016 is used. In order to stay in the vicinity of the normalized parameter values given in Eq. (4.12) the element values for Figs. 2 and 6 are chosen as follows:

$$\begin{aligned} &C_1 = 1.0\text{nF},\ C_2 = 2.0\text{nF},\ L = 2.5\text{mH},\ R = 1.7\text{K}\Omega \\ &R_a = R_b = R_d = 2.2\text{K}\Omega, R_c = 2.07\text{K}\Omega,\ R_e = 3.72\text{K}\Omega \end{aligned} \tag{5.1}$$

With the choice of above values, we are very close to the normalized parameter values given in Eq. (4.12). Specifically,

$$\begin{aligned} &\alpha = 2.0,\ \beta = 2.0645,\ \gamma = 1.0, \\ &b_3 = -1.0,\ b_1 = 0.242,\ \rho \approx 1.12. \end{aligned} \tag{5.2}$$

In this case, the frequency normalization constant is found to be $\omega_n \approx 294118$. A Hopf limit cycle obtained experimentally for the physical element values of Eq. (5.1) is shown in Fig. 7. For Fig. 7, the horizontal axis is $v_1(t)$ of Fig. 2 with 0.5V/div, and the vertical axis is $v_2(t)$ of Fig. 2 with 0.25V/div.

Our experimental simulations involve two cases: the case of a single switch

$\mathbf{S_1}$, and the case of two switches $\mathbf{S_1}$ and $\mathbf{S_2}$. To obtain rank one attractors, we choose the pulsewidth p_0 of the switch control signal such that the resulting dimensionless pulsewidth $p = 0.52$. Next, we choose and fix the values of ε_1 and ε_2. This is accomplished by fixing the values of R_1 and R_2. Then, to generate chaotic attractors, only T_0 (i.e., T) is varied. The time-T maps of the chaotic attractors obtained in the single-switch case are shown in Figs. 8 and 9. In this case, the value of R_1 is kept at 4KΩ to give $\varepsilon_1 = 0.442$. A value of $T_0 = 1/f_0 = 1/3630\text{s} \Rightarrow T = 81.0$ is used for Fig. 8, and $T_0 = 1/f_0 = 1/3060\text{s} \Rightarrow T = 96.0$ is used for Fig. 9.

In a similar manner, the time-T maps of the chaotic attractors obtained in the two-switch case are shown in Figs. 10 and 11. In this case, $R_2 = R_1 = 4\text{K}\Omega$ to give $\varepsilon_1 = 0.442$ and $\varepsilon_2 = 0.221$. A value of $T_0 = 1/f_0 = 1/3400\text{s} \Rightarrow T = 86.5$ is used for Fig. 10, and $T_0 = 1/f_0 = 1/2100\text{s} \Rightarrow T = 140$ is used for Fig. 11. In Figs. 8-11, the horizontal axis is $v_1(kT)$ of Fig. 2 with 0.5V/div, and the vertical axis is $v_2(kT)$ of Fig. 2 with 0.25V/div.

Note that the resemblance between the attractors of the numerical and the experimental simulations is striking, and that these results are in perfect agreement with the expectations of the theory [15]. It also seems that the geometric complexity of the resulting attractors increases with the number of switches employed. Another point worth mentioning here is that the width of the applied pulses p_0 is not crucial as long as it is followed by a much longer relaxation interval, i.e., $T_0 >> p_0$.

6 Concluding remarks

In this paper, we have provided the first experimental proof of rank one chaos in the smooth Chua's circuit. Our scheme involves a recipe-like procedure given by a new and comprehensive theory of dynamics, namely, the theory of rank one chaos. First, a weakly stable Hopf limit cycle coming out of a fixed point is generated. Then, under the guidance of the theory, this limit cycle is subjected to periodic kicks to obtain rank one attractors. The generic setting of the theory is satisfied by adding externally controlled switches to the original circuit. The single switch and two-switch cases have been explored experimentally. It is observed that the results of the experimental simulations are in perfect agreement with the predictions of the theory. For more background information on the theory of rank one chaos and its potential applications to circuits and systems, we refer the reader to a recent tutorial paper [15].

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FIGURES

Fig. 1.

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Fig. 6.

Fig. 7.

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Fig. 10.

Fig. 11.

FIGURE CAPTIONS

Figure 1. A switch-controlled state variable modulation scheme.

Figure 2. Switched-controlled Chua's circuit.

Figure 3. A Hopf limit cycle from numerical simulations ($\varepsilon_i = 0$).

Figure 4. A single-switch case rank one attractor from numerical simulations ($\varepsilon_1 = 0.5$, $\varepsilon_2 = \varepsilon_3 = 0$, $p = 0.5$, $T = 97.0$). (a) Phase portrait $x_k - y_k$. (b) Time evolution of x_k. (c) Frequency spectrum of x_k.

Figure 5. A two-switch case rank one attractor from numerical simulations ($\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.34$, $\varepsilon_3 = 0$, $p = 0.5$, $T = 87.5$). (a) Phase portrait $x_k - y_k$. (b) Time evolution of x_k. (c) Frequency spectrum of x_k.

Figure 6. A cubic polynomial nonlinear resistor realization.

Figure 7. A Hopf limit cycle from experimental simulations ($\varepsilon_i = 0$).

Figure 8. A single-switch case strange attractor from experimental simulations ($T = 81$, $\varepsilon_1 = 0.442$, $\varepsilon_2 = \varepsilon_3 = 0$).

Figure 9. Another single-switch case strange attractor from experimental simulations ($T = 96$, $\varepsilon_1 = 0.442$, $\varepsilon_2 = \varepsilon_3 = 0$).

Figure 10. A two-switch case strange attractor from experimental simulations ($T = 86.5$, $\varepsilon_1 = 0.442$, $\varepsilon_2 = 0.221$, $\varepsilon_3 = 0$).

Figure 11. Another two-switch case strange attractor from experimental simulations ($T = 140$, $\varepsilon_1 = 0.442$, $\varepsilon_2 = 0.221$, $\varepsilon_3 = 0$).