Problem 1.1.1(c)

Subtract twice of the first equation from the second equation and add the first equation to the third equation to get

$$\begin{cases} p & +q & -r & = 0 \\ & -3q & +5r & = 3 \\ & & -r & = 6 \end{cases}$$

The back substitution yields $r = -6$, $3q = 5r - 3 = -33$, so $q = -11$, and $p = -q + r = 5$.

Problem 1.2.3

The equality of two matrices means

$$\begin{cases} x & +y & & & = 1 \\ x & & -z & & = 0 \\ & y & & +w & = 2 \\ x & & & +2w & = 1 \end{cases}$$

Subtract the first equation from both the second and the fourth equations:

$$\begin{cases} x & +y & & & = 1 \\ & -y & -z & & = -1 \\ & y & & +w & = 2 \\ & -y & & +2w & = 0 \end{cases}$$

Add the second equation to the third equation and subtract the second equation from the fourth equation:

$$\begin{cases} x & +y & & & = 1 \\ & -y & -z & & = -1 \\ & & -z & +w & = 1 \\ & & z & +2w & = 1 \end{cases}$$

Add the third equation to the fourth equation:

$$\begin{cases} x & +y & & & = 1 \\ & -y & -z & & = -1 \\ & & -z & +w & = 1 \\ & & & 3w & = 2 \end{cases}$$

Now, by back substitution,

$$w = 2/3, \quad z = w - 1 = -1/3, \quad y = 1 - z = 4/3, \quad x = 1 - y = -1/3.$$

Problem 1.2.5(b)

The system is

$$\begin{cases} u & & +w & = -1 \\ u & +v & & = -1 \\ & v & +w & = 2 \end{cases}$$

Subtract the first equation from the second equation:

$$\begin{cases} u & & +w & = -1 \\ & v & -w & = 0 \\ & v & +w & = 2 \end{cases}$$

Subtract the second equation from the third equation:

$$\begin{cases} u & & +w & = -1 \\ & v & -w & = 0 \\ & & 2w & = 2 \end{cases}$$

The back substitution gives:

$$w = 1, \quad v = w = 1, \quad u = -1 - w = -2.$$

Problem 1.2.14

Let

$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Then

$$AB = \begin{pmatrix} a + 2c & b + 2d \\ c & d \end{pmatrix}$$

and

$$BA = \begin{pmatrix} a & c \\ 2a + b & 2c + d \end{pmatrix}.$$

$AB = BA$ if

$$\begin{cases} a + 2c = a \\ b + 2d = 2a + b \\ c = c \\ d = 2c + d \end{cases}$$

These equations imply $c = 0$, $a = d$. There are no restrictions on b.
Answer:

$$B = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$$

where a and b are arbitrary numbers.

Problem 1.3.3(c)

The augmented matrix of the system is

$$\begin{pmatrix} 2 & 1 & 2 & | & 3 \\ -1 & 3 & 3 & | & -2 \\ 4 & -3 & 0 & | & 7 \end{pmatrix}.$$

Add 1/2 times the first row to the second row and subtract twice of the first row from the second row to get

$$\begin{pmatrix} 2 & 1 & 2 & | & 3 \\ 0 & 7/2 & 4 & | & -1/2 \\ 0 & -5 & -4 & | & 1 \end{pmatrix}.$$

Add 10/7 times the second row to the third row:

$$\begin{pmatrix} 2 & 1 & 2 & | & 3 \\ 0 & 7/2 & 4 & | & -1/2 \\ 0 & 0 & 12/7 & | & 2/7 \end{pmatrix}.$$

By back substituton, $w = 1/6$. Then, $(7/2)v + (4/6) = -(1/2)$, so $(7/2)v = -(7/6)$, and $v = -1/3$. Finally, $2u - (1/3) + (2/6) = 3$, so $2u = 3$, and $u = 3/2$.