Problem 3.2.15(a)

The inner product of $(2, a, -3)^T$ and $(-1, 3, -2)^T$ equals $-2 + 3a + 6 = 3a + 4$. The vectors are orthogonal if their inner product equals 0. We get $a = -4/3$.

Problem 3.2.27

The general form of a quadratic polynomial is

$$p(x) = ax^2 + bx + c.$$

One has

$$\langle p(x), 1 \rangle = \int_{-1}^{1} (ax^2 + bx + c)dx = \frac{2}{3}a + 2c$$

and

$$\langle p(x), x \rangle = \int_{-1}^{1} (ax^3 + bx^2 + cx)dx = \frac{2}{3}b.$$

The polynomial $p(x)$ is orthogonal to both 1 and x if $b = 0$ and $2a/3 + 2c = 0$. One can take $a = 3$, $c = -1$ to get $3x^2 - 1$ as an answer. Any polynomial that is proportional to $3x^2 - 1$ is also orthogonal to 1 and x.

Problem 3.3.6(c)

To answer the question, one has to compute norms

$$||f(x) - g(x)||_\infty, \quad ||f(x) - h(x)||_\infty, \quad \text{and} \quad ||g(x) - h(x)||_\infty.$$

1. The function $f(x) - g(x) = 1 - x$ decreases from 1 to 0 on the interval $[0, 1]$, so

$$||f(x) - g(x)||_\infty = 1.$$

2. The function $\sin \pi x$ takes values from 0 to 1 on the interval $[0, 1]$, so $f(x) - h(x) = 1 - \sin \pi x$ also takes values from 0 to 1 on this interval, and

$$||f(x) - h(x)||_\infty = 1.$$

3. Both functions $g(x)$ and $h(x)$ are non-negative, and they do not exceed 1 on the interval $[0, 1]$, so $|g(x) - h(x)| \leq 1$. On the other hand, $g(1) - h(1) = 1$, so

$$||g(x) - h(x)||_\infty = \max_{0 \leq x \leq 1} |g(x) - h(x)| = 1.$$

We see that, under the L^∞ norm, the functions $f(x)$, $g(x)$, and $h(x)$ are equally distanced from each other.

Problem 3.3.28(d)

One has

$$||f(x)||_1 = \int_{-1}^{1} |x - (1/3)|dx.$$

The function $(x - 1/3)$ is positive when $x > 1/3$, and it is negative when $x < 1/3$, so

$$\int_{-1}^{1} |x - 1/3|dx = \int_{-1}^{1/3} ((1/3) - x)dx + \int_{1/3}^{1} (x - (1/3))dx = \frac{8}{9} + \frac{2}{9} = \frac{10}{9}.$$

The unit function is

$$f(x)/||f(x)||_1 = 0.9f(x) = 0.9x - 0.3.$$

Problem 3.4.22(iv)

Let $\mathbf{u} = (1, 1, 0)^T$, $\mathbf{v} = (1, 0, 1)^T$, and $\mathbf{w} = (0, 1, 1)^T$. One has

$$\langle \mathbf{u}, \mathbf{u} \rangle = \langle \mathbf{v}, \mathbf{v} \rangle = \langle \mathbf{w}, \mathbf{w} \rangle = 2$$

and

$$\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{u}, \mathbf{w} \rangle = \langle \mathbf{v}, \mathbf{w} \rangle = 1,$$

so the Gram matrix is

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

The vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linaeraly independent. In fact, if $a\mathbf{u} + b\mathbf{v} + c\mathbf{w} = 0$ then $a + b = a + c = b + c = 0$. The only solution of this system is $a = b = c = 0$. Therefore, the Gram matrix is positive definite.