

PROBLEM SET 13

PROBLEM 1

Let X be a locally compact Hausdorff topological space. The support of a function $f(x)$ is the closure of the set of all points where $f(x) \neq 0$,

$$\text{supp} f = \overline{\{x : f(x) \neq 0\}}.$$

Prove that the set of continuous functions with compact support is dense in $C_0(X)$.
Reminder: $C_0(X)$ is the space of all continuous functions $f(x)$ on X such that sets $\{x : |f(x)| \geq \epsilon\}$ are compact for all $\epsilon > 0$. The norm in $C_0(X)$ is the usual $C_b(X)$ -norm.

From Folland's book: problems 8, 9, 10, 11 on pp. 155–156; 63, 64, 65 on p. 138; 68, 69 on p. 142