PROBLEM SET 1

Problem 1

Recall that the Fourier transform of the disribution $u_\lambda = x_+^\lambda$ equals

$$\hat{u}_\lambda(\xi) = \Gamma(\lambda+1)\left[e^{-\pi i(\lambda+1)/2}\xi_+^{-\lambda-1} + e^{\pi i(\lambda+1)/2}\xi_-^{-\lambda-1}\right] \tag{1}$$

when λ is not an integer real number. Find the Fourier transform of the distribution $U_k = x_+^k$, $k = 0, 1, 2, \dots$, by taking the limit $\lambda \to k$ in (1).

Problem 2

A distribution u is called *radial* if

$$< u, \phi(Ax) >=< u, \phi(x) >$$

for every orthogonal matrix A and for every test function ϕ.
a) Find all radial distributions that are homogeneous of degree λ.
b) Prove that the Fourier transform of a radial distribution is a radial distribution.
c) Compute the Fourier transform of the distribution $u_\lambda = |x|^\lambda$.

From Evans's book: problems 15, 17, 18, p.p. 291, 292