Femtosecond soliton collapse and coherent pulse train generation in erbium-doped fiber amplifiers

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The study of the interplay of coherent and Raman effects on femtosecond soliton collapse and pulse train generation in erbium-doped fiber amplifiers is examined.

The amplification of ultrashort pulses in laser amplifiers is a well established field of physics and technology.[1] Nevertheless, it appears that the recent rapid development of doped fiber amplifiers and lasers has opened up, to the experimenter and to the physicist, a new branch of this field.[2] The possibilities that doped fiber amplifiers offer for the in-line amplification of optical solitons may provide a significant breakthrough in the future of long-distance telecommunications.[3]

Recent experiments[4–6] have reported the amplification of femtosecond solitons in erbium-doped optical fibers. The spectral width of these pulses is comparable to the linewidth of the amplifier. As a consequence, the soliton sees a reduced gain with respect to a weak continuous wave signal. However, the observations showed that the pulse width may remain unchanged over some initial amplification distance. In fact, the pulse shape is determined from a balance between two opposite effects. From one side, the nonlinear index and dispersion of glass would compress the time width of a soliton of an order higher than one. On the other hand, the finite bandwidth of the gain favors the amplification of the central portion of the soliton spectrum, which would broaden the pulse.

The temporally undistorted pulse that may result from a compensation between these two effects is an unstable phenomenon.[7] In fact, in Refs. 5 and 6 it was observed that the initial undistorted soliton amplification is followed by a relatively sudden temporal compression, that is accompanied by substantial spectral reshaping. In this letter we analyze this soliton collapse effect, that is activated by self-phase modulation along the pulse profile which breaks the uniformity of the soliton phase. As we shall see, the collapsed soliton evolves into a train of pulses that collide in the presence of Raman self-scattering.[8,9]

Soliton propagation in the presence of finite bandwidth gain has been studied by introducing linear gain and gain dispersion in the nonlinear Schrödinger (NLS) equation.[6,7,10] Clearly, this approximation is only valid when the spectral width of the pulse is much narrower than the amplifier bandwidth. This condition is not satisfied for pulse widths of 200 fs or shorter.[4–6] Therefore we take into account here, the coherent interaction between the field and the doping atoms,[11,12] as well as the effects of self-stimulated Raman scattering in the fiber.

The coupled equations for the single mode electric field E, the polarization P, and the population inversion W of a homogeneously[13] broadened system of two-level atoms in the active fiber read, in the slowly varying envelope approximation,

$$\frac{\partial E}{\partial Z}+\beta'\frac{\partial E}{\partial T}+\frac{i\beta''}{2}\frac{\partial^2 E}{\partial T^2} = iRE\left(\rho\left|E\right|^2+(1-\rho)\int_{-\infty}^{T}\left|E(\tau)\right|^2 f(T-\tau)d\tau\right)+\alpha' P,$$

$$\frac{\partial P}{\partial T}=-\left(\frac{1}{T_2}+i\Delta\right)P+\frac{\wp}{\hbar}EW,$$

$$\frac{\partial W}{\partial T}=-\frac{\wp}{2\hbar}(PE^*+EP^*)-\frac{W-W_0}{T_1}, \tag{1}$$

where $1-\rho\simeq 0.2$ is the fractional Raman contribution to the nonlinear index, $R=\omega n_2/c$, $\Delta=\omega_0-\omega$, $\omega_0=(E_a-E_b)/\hbar$ is the transition frequency, $\alpha' = N\wp\mu_0\omega_0^2/2\beta$, β, β', and β'' are the modal propagation constant and its derivatives with respect to frequency. Furthermore,

$$f(T)=\frac{\tau_1^2+\tau_2^2}{\tau_1\tau_2^2}e^{-T/\tau_2}\sin(T/\tau_1) \tag{2}$$

whose Fourier transform is a Lorentzian fit to the complex Raman susceptibility of optical fibers, where $\tau_2 = 32$ fs and $\tau_1 = 12.2$ fs.[7] Moreover, T_1 and T_2 are the population and polarization decay times, and W_0 is the equilibrium inversion that results from the radiative decay from pumping into an upper metastable level, and $\wp$ is the effective dipole moment of the emitting transition.

Note that in writing Eqs. (1) and (2) we have made several simplifying assumptions. In fact, erbium in glass is a rather complex multilevel system where the upper and lower levels exhibit Stark splitting into several sublevels. As a result, the frequency dependent nonlinear susceptibility of erbium changes with the pump power: the gain profile exhibits two spectral peaks of variable relative height.[2] The amplification of the signal pulse will also depend on the transverse doping distribution along the fiber core. Moreover, the detailed frequency structure of the Raman gain, as well as its dependence on the doping concentration is neglected in Eqs. (1) and (2). Whenever the amplification of the femtosecond soliton pulse along the fiber does not significantively change the initial inversion,[12] Eqs. (1) and (2) basically involve approximating with a Lorentzian the frequency dependent gains that are associated with the

Raman and Erbium transitions. As we shall see, this is justified by the good qualitative and quantitative agreement between the predicted and observed dynamics of the soliton amplification process.

For a numerical solution of Eq. (1), we make a change to the dimensionless variables

$$\frac{\wp\tau_0 E}{\hbar}\to u,$$
$$(T-Z\beta')\Big/\tau_0\to t,\quad \frac{Z}{Z_0}\to z,\quad \frac{T_{1,2}}{\tau_0}\to T_{1,2}$$
$$\frac{\tau_{1,2}}{\tau_0}\to\tau_{1,2},\quad \frac{Z_0\beta''}{\tau_0^2}\to\beta'',\quad \frac{W}{W_0}\to W, \tag{3}$$
$$\frac{Z_0 R\hbar^2}{\wp^2\tau_0^2}\to R,$$

where τ_0 is an arbitrary input pulse width, and

$$Z_0 \doteq \frac{\hbar}{\alpha'\wp\tau_0 W_0}. \tag{4}$$

One obtains

$$\frac{\partial u}{\partial z}+\frac{i\beta''}{2}\frac{\partial^2 u}{\partial t^2}=iRu\left(\rho\left|u\right|^2+(1-\rho)\times\int_{-\infty}^{t}\left|u(\tau)\right|^2 f(t-\tau)\,d\tau\right)+P,$$

$$\frac{\partial P}{\partial t}=-\frac{P}{T_2}+Wu,$$
$$\frac{\partial W}{\partial t}=-\frac{1}{2}(Pu^*+uP^*), \tag{5}$$

where we assumed that $\tau_0 \ll T_1$.

We have used the boundary conditions

$$E(z=0,t)=A_0\sqrt{\beta''/R}\,\mathrm{sech}(t),$$
$$P(z,t=-T)=0,\quad W(z,t=-T)=1, \tag{6}$$

where T is the half width of the computational temporal window. Note that for $A_0=1$ the input pulse is a $N=1$ soliton of the NLS equation. Figures 1 and 2 show the calculated pulses and spectra for two different fiber lengths in conditions similar to the experiments in Ref. 6. We have set the fiber dispersion equal to $D=-5$ ps/nm km, and the input time width $\tau_0=142$ fs, that yields a full width at half maximum of 250 fs. The homogeneous linewidth of the transition at $\lambda=1.53$ μm of erbium is 10 nm,[13] that leads to $T_2\simeq 250$ fs. The doping concentration was $N=5\times10^{18}$ cm^{-3}, whereas the erbium dipole moment is $\wp=2.5\times10^{-20}$ esu.

In Figs. 1 and 2 we report the output pulse profiles and spectra after 1.5 and 3 m of active fiber, respectively. The relatively large linear gain $g(m^{-1}) = N\wp^2\mu_0\omega_0^2T_2W_0/(2\beta\hbar) = T_2/(\tau_0 z_0)$ is 4.2 dB/m, which involves nonadiabatic soliton amplification. Whereas $A_0=1.4$ in the numer-

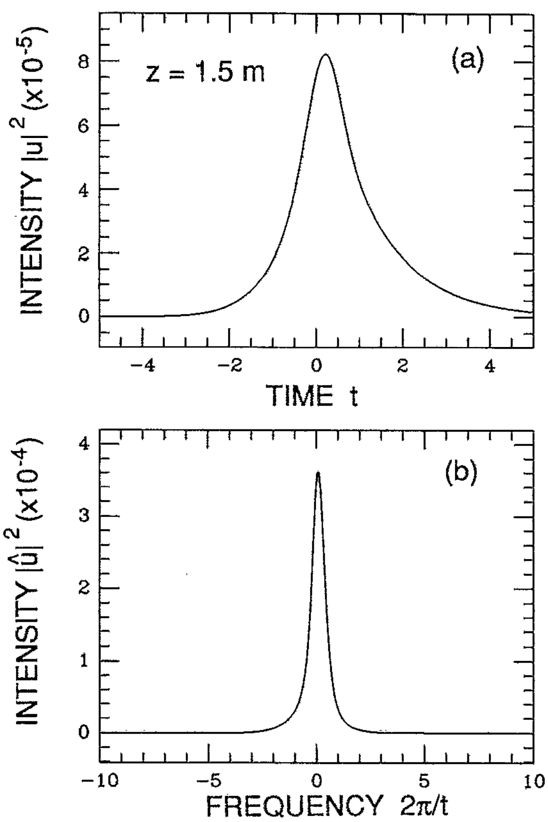

FIG. 1. Calculated pulse intensity $|u|^2$ (a) and spectral intensity $|\hat{u}|^2$ (b) from 1.5 m of active fiber with 4.2 dB/m linear gain.

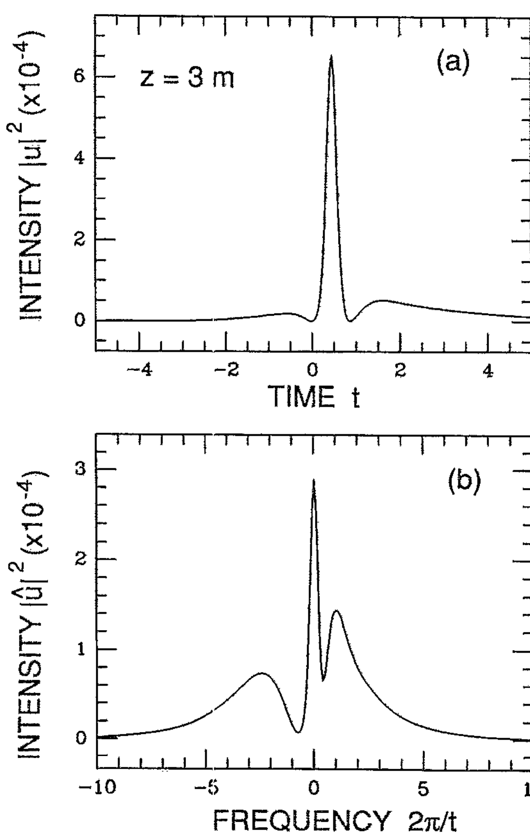

FIG. 2. As in Fig. 1: pulse (a) and spectrum (b) from 3 m of active fiber.

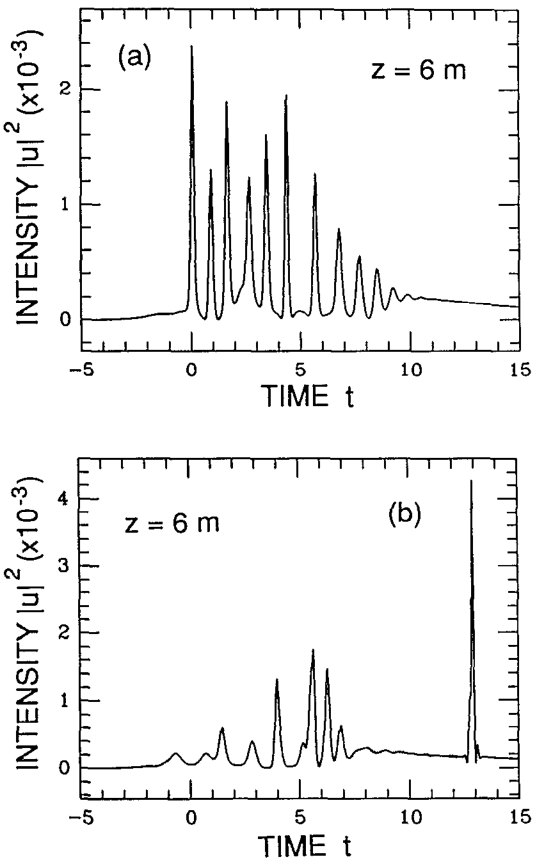

FIG. 3. As in Fig. 1 with a fiber length of 6 m, and 7 dB/m linear gain: (a) without Raman effect; (b) with Raman effect.

ical solution of Eqs. (5) and (6), including the Raman effect. Note that the linear gain is calculated from the complex Lorentzian susceptibility that one obtains by setting $\partial W/\partial T = 0$ in Eq. (1). The dimensionless units of Eq. (5) are reported in the figures. Figure 1 shows that in the initial part of the propagation the pulsewidth has remained almost unchanged, whereas the spectral width is slightly narrower than the input value. On the other hand, Fig. 2 shows that after a certain distance (that is inversely proportional to the linear gain) the amplified input pulse rapidly narrows in time down to about 40 fs. In the frequency domain, the signature of this soliton compression is the appearance of three distinct spectral peaks. The central peak is associated with the long tail at the trailing edge of the pulse and increases when T_2 grows larger, whereas the two asymmetric sidebands originate from the self-phase-modulated narrow pulse. The soliton compression and spectral reshaping in Fig. 2 is due to the interplay between the Kerr effect and the finite bandwidth of gain and dispersion. The compression enhances the soliton self-frequency shift (SSFS),[8,9] which leads to the escape of the soliton from the amplification line.[5,6]

Figure 3 clarifies the effects of Raman scattering: here the fiber is 6 m long and the gain is 7 dB/m. Figure 3(a) was obtained with $\rho = 1$ in Eq. (1), whereas in Fig. 3(b) the Raman term is included. Neglecting the Raman effect, one obtains a train of pulses at the fiber output [Fig. 3(a)]. These are not independent solitons: rather, the train may be thought as a chain of infinitely many coupled solitons, in analogy with the self-similar pulse trains that are generated by a two-level amplifier .[14] Owing to SSFS, the leading pulse of the train gains energy from the multiple inelastic collisions with the other pulses [see Fig. 3(b)]. This single amplified soliton slows down and eventually it separates in time from the broad background.

In conclusion, we have shown that a simple two-level atom model coupled with glass dispersion, Kerr and Raman effects may well reproduce the observed characteristics of the amplification of femtosecond pulses in erbium-doped fibers. Moreover, we predict the generation of a pulse train that collide under the influence of SSFS until a single amplified and compressed soliton emerges from the fiber.

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