Recovery of solitons with nonlinear amplifying loop mirrors

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We study the use of nonlinear amplifying loop mirrors to recover soliton pulses nonadiabatically deformed by losses. We approach this problem as a mapping problem of input pulse to output pulse, for segments of fiber followed by a combination of linear and nonlinear amplification. For a wide range of amplifier spacings, we find numerically that a single optimal input pulse of soliton shape exists for each amplifier spacing, which is well recovered at output. The recovered output pulses contain only ~3% continuous radiation. © 1995 Optical Society of America

The problem of optical soliton transmission divides into two categories: (1) long-distance transmission based on dispersion-shifted fiber with periodic amplification and (2) the improvement of existing networks based on standard monomode fibers (SMF's). In the former, center-guiding soliton transmission can be used.[1] In the latter the dispersion length is of the same order as the amplifier spacing for desired bit rates; thus dispersion is very significant, and center-guiding soliton transmission is not applicable. Recent investigations[2] indicate that the capacity of existing SMF networks operating in the 1.55-μm window may be improved by the so-called oversoliton method.

In this paper we study the use of nonlinear amplifying loop mirrors (NALM's) introduced by Fermann *et al.*[3] as an alternative to the oversoliton method to recover the soliton profile of nonadiabatically deformed pulses in SMF networks. Previous studies[3–5] have addressed the use of the NALM to reshape pulses.

We report that fixed-point solutons of the pulse mapping, generated by propagation in lossy fibers followed by a NALM plus some additional linear amplification, can be approximated very well with particular soliton input pulses of the form $\eta\ \mathrm{sech}(\eta T)$ for a wide range of amplifier spacings and fiber losses.

In standard soliton units the evolution of pulses in monomode optical fibers is well described by the damped nonlinear Schrödinger equation[6]

$$\frac{\partial E}{\partial \xi} = \frac{i}{2}\frac{\partial^2 E}{\partial \tau^2} + i|E|^2 E - \Gamma E\,, \tag{1}$$

where E is the complex envelope of the electric field, ξ is the fiber coordinate, and τ is the retarded time.

For applications we must be able to consider solutions of Eq. (1) for the initial condition $E_0 = \beta\ \mathrm{sech}(\beta\tau)$ with arbitrary values of β, Γ, and amplifier spacings ξ_a. Of these three parameters, Γ and ξ_a are dictated by applications. With rescaling by

$$Z \equiv \xi/\xi_a, \quad \Gamma \equiv \tau/\sqrt{\xi_a}, \quad q \equiv E\sqrt{\xi_a}, \quad \gamma \equiv \xi_a \Gamma\,,$$

Eq. (1) becomes

$$\frac{\partial q}{\partial Z} = \frac{i}{2}\frac{\partial^2 q}{\partial T^2} + i|q|^2 q - \gamma q\,. \tag{2}$$

Also, defining $\eta \equiv \beta\sqrt{\xi_a}$, E_0 becomes $q_0 = \eta\ \mathrm{sech}(\eta T)$. The amplifier spacing is now scaled to 1, leaving only γ and η free. Given z_a in kilometers and fiber losses L_{dB} in decibels per kilometer, we have $\gamma = (z_a L_{\mathrm{dB}} \log 10)/[20(\mathrm{km})]$. The pulse width t_s in picoseconds is $1.76\sqrt{z_a k''}/\eta$, where k'' is the group-velocity dispersion in picoseconds squared per kilometer. For a fixed fiber type there is a one-to-one correspondence between γ and z_a, and also between η and t_s for z_a fixed.

If the nondimensional NALM loop length L, the loop amplifier gain G, and the input amplitude η are such that $\eta^2 L(G \pm 1)/4 \ll 1$, then the NALM approximately cubes the amplitudes of pulses[4]:

$$q_{\mathrm{out}} = iA|q_{\mathrm{in}}|^2 q_{\mathrm{in}}\,, \tag{3}$$

where

$$A \equiv \sqrt{G_{\mathrm{add}}}\,\frac{L\sqrt{G}(G-1)}{4}\,. \tag{4}$$

Here G_{add} represents the gain of an additional amplifier following the NALM, which is sometimes necessary for finding particular values of A while maintaining the cubing effect.

We report fixed points of the mapping problem with a cubing NALM. We found the points by fixing γ and numerically simulating pulse propagation and amplification by the NALM for different values of η to find the optimal input pulse. The parameter A is

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chosen for each trial value of η such that the output pulse also has peak amplitude η.

Following Matsumoto *et al.*,[4] we define the optimal pulse to be the one that produces the least percentage of dispersive wave energy and the least-dispersive components in the nonlinear Schrödinger Hamilitonian in the output pulse. These percentages are labeled ΔE and ΔH, respectively. Useful expressions for these quantities appear in Matsumoto *et al.*[4]

We find that for each choice of γ in the interval [0.3, 1] there is a unique optimal pulse with a minimum dispersive energy output of roughly 2% or 3%, with a minimum of dispersive components of the Hamiltonian between 0.3% and ~6%. Outside this range in γ, or for choices of η smaller or larger than optimal, the integrity of the output pulse is significantly degraded.

The optimal values of A and η are given in Table 1 and Fig. 1(a). The percentages ΔE and ΔH are given in Fig. 1(b). The optimal values of η do not depend strongly on γ and therefore not strongly on amplifier spacing, which may be important for applications.

We emphasize that the optimal pulses reported here do not decay adiabatically during the propagation stage and are therefore not an example of the adiabatic amplification of solitons reported by Matsumoto *et al.*[4] Adiabatic decay occurs only at values of η significantly higher than the optimal values that we report, and it can be shown that the adiabatic approach can solve the full mapping problem that we pose here only at $\gamma \approx 0.33$. The fact that the optimal values of η fall below the adiabatic regime is somewhat advantageous because the optimal intensities are not so high as to invalidate the damped nonlinear Schrödinger equation model that we are using.

Because of the NALM cubing effect, the optimal pulses are intrinsically unstable under repeated recoveries by the NALM unless an additional technique is used stabilize the absolute and the relative amplitudes of pulses. If the propagation stage were perfectly linear, it follows by linearizing Eq. (3) that small amplitude perturbations would grow by exactly a factor of 3 uniformly in time at each NALM. In reality, the propagation stage operates on perturbations in a temporally nonlocal way that is difficult to characterize concisely for all perturbations. For the case of parametric perturbations, i.e., for initial pulses for the form $q_0 = (\eta + \delta\eta)\text{sech}(\eta + \delta\beta)\text{T}$, an estimate for the mapping of $\delta\eta$ and $\delta\beta$ from NALM to NALM is

$$\begin{bmatrix} \delta\eta_{n+1} \\ \delta\beta_{n+1} \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} \delta\eta_n \\ \delta\beta_n \end{bmatrix}. \qquad (5)$$

This map can be inferred because, in the undamped nonlinear Schrödinger equation, an initial condition of the form $q_0 = (\eta + \delta\eta)\text{sech}(\eta + \delta\beta)T$ produces a soliton with amplitude $(\eta + 2\delta\eta - \delta\beta)\text{sech}(\eta + 2\delta\eta - \delta\beta)T$. The eigenvalues of the matrix of the right-hand side of Eq. (5) are 5 and 0, with the eigenvector associated with 5 located along the line $\delta\beta = \delta\eta/3$ (the actual eigenvalues for $0.3 \le \gamma \le 1$ were numerically found to lie within the ranges 3.5 to 5 and -0.35 to -0.20, respectively). As we verified in simulations, these eigenvalues imply that the perturbation will decrease initially at each NALM if $(2\delta\eta_n - \delta\beta_n)$ is sufficiently small but thereafter will grow roughly along the line $\delta\beta = \delta\eta/3$ by a factor of 5 at each NALM. Therefore propagation effects can initially hinder but ultimately aggravate the instability. This map also predicts a tendency for pulses just below optimal amplitude to broaden in pulse width as they decay, whereas pulses just above become narrower as they diverge from the optimal pulse shape.

Optimal Values of A and η

γ	A	η
0.3	0.4	3.97
0.4	1.2	2.73
0.5	2.1	2.37
0.6	3.3	2.19
0.7	5.0	2.05
0.8	7.4	1.95
0.9	10.7	1.88
1.0	15.3	1.83

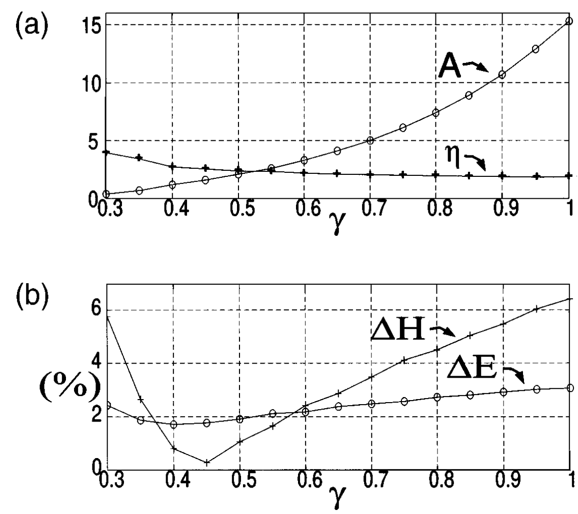

Fig. 1. (a) Optimal parameters, (b) dispersive percentages.

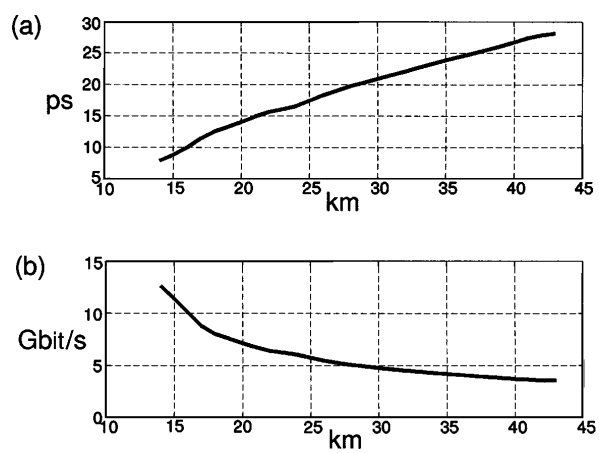

Fig. 2. (a) Optimal pulse widths, (b) optimal bit rates as functions of amplifier spacing.

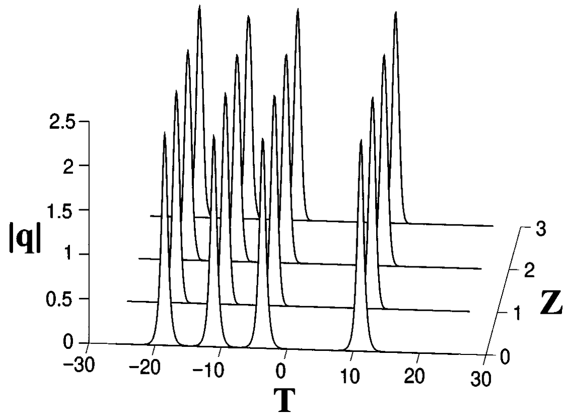

Fig. 3. Optimal bit stream undergoing several successful recoveries.

In numerical simulations, indefinitely stable recovery of *single* pulses can be achieved by a slight adjustment of the parameter A at each NALM to maintain the peak output amplitude equal to the optimal η. This suggests that residual continuous radiation does not aggravate the instability for single pulses. However, adjustment of A at each NALM is not practical and also will not prevent the instability from widening relative differences between the peak amplitudes of pulses. Numerical simulations of bit streams with interacting pulses (5 to 10 pulse widths apart) suggest that distances of 50 to 200 km for SMF with low error rates might be achievable without additional stabilization measures. Bit rates versus amplifier spacing for SMF at 1.55 μm for the optimal pulses with 10-pulse-width spacing are shown in Fig. 2. Figure 3 shows snapshots from a numerical simulation of a bit stream of optimal pulses at $\gamma = 0.5$ undergoing several successful recoveries without any stabilization measures (the pulses are shown after each NALM).

As an example of optimal NALM parameters for SMF, choosing an amplifier spacing of 21.7 km yields $\gamma = 0.5$, $A = 2.1$, and $\eta = 2.37$. A choice of NALM parameters that produces this value of A while preserving the cubing approximation to high accuracy is $G = G_{\text{add}} = 236$ (23.7 dB) with a loop length of 3.3 m.

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