Averaged pulse dynamics in a cascaded transmission system with passive dispersion compensation

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A theory of optical pulse propagation in cascaded transmission systems that are based on the dispersion-compensating fiber technique is developed. The existence of two scales associated with fiber dispersion and system residual dispersion leads to a simple model for the averaged pulse dynamics. In the particular case of practical importance, the averaged pulse dynamics is governed by the nonlinear Schrödinger equation. The pulse transmission stability is examined.

The application of dispersion-compensating fibers (DCF's) to overcome fiber chromatic dispersion in optical transmission systems has been the subject of intensive investigations during the past few years.[1–6] This approach is very promising inasmuch as it is simple. It is compatible with the present concept of the all-optical transparency of the system and is cascadable. Most investigations to date focused primarily on point-to-point transmission lines operating in a linear regime. They showed the great potential of the dispersion-compensating method, especially taking into account its high-capacity, low-error bit rate and large amplifier spacing. However, the theory of optical pulse propagation in such systems is not developed. Here we present a model describing optical pulse dynamics in the cascaded transmission system based on DCF. As a result of our research, the new concept of breathing solitons is introduced.

A cascaded transmission system containing optical amplifiers was investigated in recent experiments.[4,5] In this research a 40-Gbit/s nonreturn-to-zero bit stream was sent over 600 km, and an 80-Gbit/s bit stream was transmitted over 400 km with a bit-error rate below 10^{-11}. In this Letter we use the system design described in Refs. 4 and 5 as a typical example of a cascaded system based on the dispersion-compensating technique.

Let us examine a transmission line composed of periodic alternating fiber sections and lumped optical amplifiers. Each section includes a piece of DCF with normal dispersion D_{DCF} and pieces of fiber with anomalous dispersion D_{TF}, as shown in Fig. 1. Ideally, the dispersions should be fully compensated; however, in practice there is always some residual dispersion. For example, in the experiment presented in Refs. 4 and 5, the residual dispersion was approximately equal to $\langle |D| \rangle \sim 0.5$ ps/(nm km), whereas dispersion of the transmission fiber was $D \simeq +(16-18)$ ps/(nm km). Thus for the system we can identify three characteristic dispersion scales: the dispersion length Z_{DCF}, corresponding to the chromatic dispersion $D_{\text{DCF}} \simeq -(65-80)$ ps/(nm km) of the DCF; the dispersion length of the transmission fiber, Z_{dis}; and the dispersion length corresponding to the residual dispersion of each section, Z_{DR}. Therefore, in the first approximation, pulse propagation in the transmission line can be described as follows (see Fig. 1): During the first stage, propagation through the DCF, pulses broaden dispersively and acquire a positive dispersion-induced frequency chirp $C > 0$; in the second stage—evolution through the transmission fiber—pulses compress because the sign of the dispersion has been reversed and the condition for compression $\beta_2 C < 0$ is satisfied (see, e.g., Ref. 7). Thus the pulse experiences breatherlike oscillations. During both stages the amplitude of the propagating

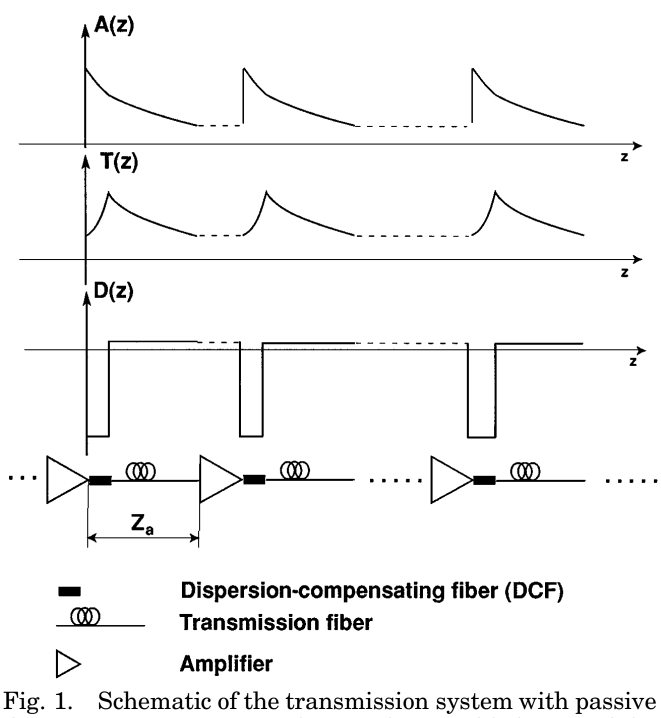

Fig. 1. Schematic of the transmission system with passive dispersion compensation along with typical behavior of the pulse amplitude $A(z)$, pulse width $T(z)$, and dispersion $D(z)$ along the fiber transmission line.

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pulse is reduced because of fiber losses. Therefore after pulses propagate through both pieces of fiber they must be amplified by an optical amplifier. This entire process, including amplification, is then repeated many times. The Kerr nonlinearity over one cycle of this process is negligibly small; therefore, dispersion and fiber losses are the main factors. The influence of the residual dispersion appears at a distance of $Z_{\rm DR}$ and alters the shapes of pulses. For the transmission system to have a sufficiently small bit-error rate, the amplitude of the incident pulse must not be too small; therefore at distances of the order of $Z_{\rm DR}$ the influence of Kerr nonlinearity might come into play. Thus, in the description of a slow evolution of a pulse, it is necessary to take into account not only the residual dispersion but the nonlinearity as well.

Let us introduce the following characteristic scales into this problem: Z_a, the amplification period; $Z_{\rm dis} = 2t_0^2/|\beta_2|$, the dispersion length corresponding to the standard monomode fiber (SMF) (as mentioned above, in the system under consideration there exist, in fact, several dispersion lengths); and $Z_{\rm NL} = 1/(\alpha P_0)$, the nonlinear length. Here t_0 and P_0 are the incident pulse width and peak power, β_2 is the group-velocity dispersion, and α is the nonlinear coefficient. The evolution of optical pulses in a fiber is described by the nonlinear Schrödinger equation:

$$iE_z + \frac{Z_{\rm NL}}{Z_{\rm dis}} d(z)E_{tt} + |E|^2E = iG(z)E\,,$$

$$G(z)E = Z_{\rm NL}\left[-\gamma + r\sum_{k=1}^{N}\delta(z - z_k)\right]E\,. \qquad (1)$$

Here $E = E(t, z)$ is an envelope of electric field normalized by $\sqrt{P_0}$; t and z are the time and the coordinate along the fiber, normalized by t_0 and $Z_{\rm NL}$, respectively; γ describes fiber losses; $z_k = kZ_a/Z_{\rm NL}$ ($k = 1, \ldots, N$) are the amplifier locations; and r is the coefficient of amplification that compensates the fiber losses over the distance Z_a. Chromatic dispersion $d(z)$ is normalized to the SMF dispersion coefficient.

We would like to point out a difference between the problem that we consider and a soliton propagating in the ultralong communication systems with varying gain and dispersion studied in Refs. 8 and 9. Note that, in the case of a guiding-center soliton, the evolution of the pulse between the amplifiers also can be described by linear theory. The difference is that in the case that we analyze here ($Z_{\rm NL} \gg Z_a, Z_{\rm dis}$), *both the dispersion and the losses affect the pulse propagation.* In the case of the guiding-center soliton only the fiber losses are significant (the dispersion and the nonlinearity can be treated as perturbations), causing the amplitude oscillations, while the form of the pulse remains unchanged. In the guiding soliton concept the slow changes occurred under the influence of nonlinearity and dispersion at distances of the order of the soliton period ($Z_a \ll Z_{\rm NL} \sim Z_{\rm dis}$).

Note that ratios $\varepsilon_1 = Z_{\rm dis}/Z_{\rm DR}$ and $\varepsilon_2 = Z_a/Z_{\rm NL}$ are two independent small parameters of the problem. The existence of these small parameters allows us to decompose the solution of Eq. (1) into the sum of slowly and rapidly varying functions. Following Refs. 8 and 9, we transform E into a new function q by taking out rapid oscillations of the amplitude that are due to periodic amplification $E = q(t, z)\exp[\int_0^z G(z')\mathrm{d}z']$. The equation for q reads as

$$iq_z + \frac{Z_{\rm NL}}{Z_{\rm dis}} d(z)q_{tt} + c(z)|q|^2q = 0\,. \qquad (2)$$

Here $c(z) \equiv \exp[2\int_0^z G(z')\mathrm{d}z']$ can be presented as a sum of rapidly varying and constant parts $c(z) = \langle c(z)\rangle + \tilde{c}(z)$, where $\langle \tilde{c}(z)\rangle = 0$ and $\langle \tilde{c}(z)\rangle = [1 - \exp(-2\gamma z_a)]/(2\gamma z_a)$. We also write $d(z)$ in a similar form, $d(z) = \langle d(z)\rangle + \tilde{d}(z)$, where $\langle \tilde{d}(z)\rangle = 0$ and $\langle \tilde{d}(z)\rangle \approx Z_{\rm dis}/Z_{\rm DR} \ll 1$. The average value is defined here as $\langle f\rangle = 1/z_a \int_0^{z_a} f(z)\mathrm{d}z$.

In the limit $Z_a, Z_{\rm dis} \ll Z_{\rm NL}$, one may treat the nonlinearity as a perturbation. In the lowest order, the solution of Eq. (2) is $q(z, t) = \int_{-\infty}^{+\infty}\mathrm{d}\omega q_\omega \exp[i\omega t - i\omega^2(Z_{\rm NL}/Z_{\rm dis})\int_0^z \tilde{d}(\xi)\mathrm{d}\xi]$. Here q_ω does not depend on z. Nonlinear effects come into play on a larger scale than with Z_a, namely, at the distances proportional to $Z_{\rm NL}$. Therefore to describe the influence of the nonlinearity on the pulse propagation we assume that q_ω varies slowly with z. The evolution of $q_\omega(z)$ is governed by the equation

$$i\,\frac{\partial q_\omega(z)}{\partial z} - \omega^2\frac{Z_{\rm NL}}{Z_{\rm dis}}\langle d(z)\rangle q_\omega(z) + c(z)Q(z) = 0\,, \qquad (3)$$

$$Q = \int_{-\infty}^{+\infty}\mathrm{d}\omega_1\mathrm{d}\omega_2\mathrm{d}\omega_3 q\omega_1 q\omega_2 q\omega_3^*\delta(\omega_1 + \omega_2 - \omega_3 - \omega) \times \exp\left[i(\omega^2 - \omega_1^2 - \omega_2^2 + \omega_3^2)\frac{Z_{\rm NL}}{Z_{\rm dis}}\int_0^z \tilde{d}(\xi)\mathrm{d}\xi\right]. \qquad (4)$$

To obtain an equation for the slow evolution of $q(t, z)$, we average Eq. (3) over the interval Z_a. Since the function $q(\omega, z)$ is assumed to vary slowly on the amplification distance, it can be placed outside the averaging integral. After straightforward calculations we obtain the following equation:

$$i\,\frac{\partial q_\omega}{\partial z} - \omega^2\frac{Z_{\rm NL}\langle d\rangle}{Z_{\rm dis}} q_\omega + \int_{-\infty}^{+\infty}\mathrm{d}\omega_1\mathrm{d}\omega_2\mathrm{d}\omega_3\delta(\omega_1 + \omega_2 - \omega - \omega_3) \times F(\omega_1, \omega_2, \omega_3, \omega)q_{\omega_1}q_{\omega_2}q_{\omega_3}^* = 0\,, \qquad (5)$$

with the function F given by

$$F = \frac{1 - \exp[-2\gamma Z_c(1 - igA)]}{2\gamma(1 - igA)} + \frac{\exp[-2\gamma Z_c(1 - igA)] - \exp[-2\gamma Z_c(1 + ig\langle d\rangle)]}{2\gamma(1 + ig)}\,, \qquad (6)$$

where $g = (\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega^2)/(2\gamma Z_{\rm dis})$ and $A = |D_{\rm DCF}/D_{\rm TF}|$. The principal results of our Letter are based on Eq. (5). This equation describes averaged dynamics of breathing pulses. Let us consider the evolution of a localized pulse. By normalizing the frequency in Eq. (5) to the characteristic spectral pulse width $\omega_{\rm cr}$ [here $\omega_{cr}^2 = Z_{\rm dis}/(Z_{\rm NL}\langle d\rangle) \sim Z_{\rm DR}/Z_{\rm NL}$], we find that $g \sim \omega_{\rm cr}^2/(\gamma Z_{\rm dis})$. In the limit $\omega_{\rm cr}^2/(\gamma Z_{\rm dis}) \ll 1$ one can approximate the function F by the first terms of the Taylor expansion ($g \ll 1$). In the time domain, the leading-order equation describing the averaged dynamics of breathing solitons is

$$iU_z + \frac{Z_{\rm NL}}{Z_{\rm dis}} U_{tt}\langle d(z)\rangle + \langle c(z)\rangle |U|^2 U = R_2 \, . \tag{7}$$

Here $U = U(t, z) = \int_{-\infty}^{+\infty} \exp(i\omega t) q_\omega(z) {\rm d}\omega$ it the slowly varying part of the field E. In Eq. (7) the term R_2 describes higher-order corrections. To derive higher-order corrections to the averaged nonlinear Schrödinger equation [left-hand side of Eq. (7)] one can use either the Lie transformation technique[8] or a combination of the approach exploited in Ref. 9 and the procedure developed in Ref. 10. In the leading order, Eq. (7) has the usual soliton solution $U(t, z) = U_0 \,{\rm sech}(t/\tau)\exp(iz/2)$, with $U_0^2\langle c\rangle = 1$ and $\tau^2 = 2\langle d\rangle Z_{\rm NL}/Z_{\rm dis}$, if the residual dispersion $\langle d(z)\rangle$ has a positive sign. Therefore the sign of the residual dispersion $\langle d(z)\rangle$ in Eq. (7) represents a criterion for the stability of the data stream.

The amount of continuous radiation generated during the propagation of a breathing soliton and its dependence on system parameters can be determined by the higher-order correction of Eq. (7). We will relegate the discussion of these corrections to another publication. Equation (7) leads to the following algorithm of optimal dispersion management: $\sum_{j=1}^{N} (\langle d\rangle_j - \langle d\rangle)^2 \rightarrow$ min with respect to all possible permutations of DCF and SMF components. Here $\langle d\rangle_j$ is the residual dispersion of the jth transmission element and $\langle d\rangle$ is the normalized mean value of the whole system dispersion. This algorithm is in many ways similar to the algorithm proposed in Ref. 11 and provides the minimal variation of dispersion. The case that we considered here is of practical importance. The values of parameters corresponding to the pulse width $t_0 \sim 25$ ps and the peak power $P_0 \sim 2$ mW are $Z_a \sim 36$ km, $Z_{\rm dis} = 2t_0^2/|\beta_2| \sim 73$ km, and $Z_{\rm NL} = (\alpha P_0)^{-1} = 300$ km, where $\beta_2 = -20$ ps^2/km and $\alpha = 1.66$ km^{-1} W^{-1} are typical for conventional silica fibers at $\lambda_0 = 1550$ nm.[7] We take the value of the dispersion length corresponding to residual dispersion as $Z_{\rm DR} \sim 200$ km.

In conclusion, we have shown that breathing solitons can propagate in cascaded transmission lines with periodic lumped amplification and periodic dispersion compensation if $Z_{\rm NL} \gg Z_a, Z_{\rm dis}$. In the span between two amplifiers, a pulse experiences strong attenuation and large oscillations of its width. We have derived the equation describing propagation of averaged pulses in such systems. In the limit $\gamma Z_{\rm dis} Z_{\rm NL}/Z_{\rm DR} = \gamma D_R/(\alpha D_{\rm TF} P_0) \gg 1$ this propagation can be described by the nonlinear Schrödinger equation. Note that the pulse width does not enter into the final form of our criterion; therefore the theory is applicable to both return-to-zero and nonreturn-to-zero signal formats. Our results complement the concept of guiding-center solitons and demonstrate that the concept of averaged dynamics can be applied in this modification of the original system. This is a further confirmation of the robust nature of optical solitons.

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References

1. H. Onaka, H. Miyta, K. Otsuka, and T. Chikama, in *Proceedings of European Conference on Optical Communication '94* (Instituto Internazionale delle Communicazione, Genova, Italy, 1994), Vol. 4, p. 49; A. D. Ellis and D. M. Spirit, Electron. Lett. **30,** 72 (1994).
2. C. D. Chen, J.-M. P. Delavaux, B. W. Hakki, O. Mizuhara, T. V. Nguyen, R. J. Nuyts, K. Ogawa, Y. K. Park, R. E. Tench, L. D. Tzeng, and P. D. Yeates, Electron. Lett. **30,** 1159 (1994).
3. A. D. Ellis and D. M. Spirit, Electron. Lett. **30,** 72 (1994).
4. C. Das, U. Gaubatz, E. Gottwald, K. Kotten, F. Küppers, A. Mattheus, and C. J. Weiske, Electron. Lett. **31,** 305 (1995).
5. C. Das, B. Hein, U. Gaubatz, E. Gottwald, K. Kotten, F. Küppers, A. Mattheus, L. Rapp, and C. J. Weiske, Electron. Lett. **31,** 1374 (1995).
6. F. M. Knox, W. Forysiak, and N. J. Doran, J. Lightwave Technol. **13,** 1955 (1995).
7. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, San Diego, Calif., 1989).
8. A. Hasegawa and Y. Kodama, Opt. Lett. **15,** 1444 (1990); Phys. Rev. Lett. **66,** 161 (1991).
9. L. F. Mollenauer, S. G. Evangelides, Jr., and H. A. Haus, J. Lightwave Technol. **9,** 194 (1991).
10. Y. S. Kivshar, K. H. Spatschek, M. L. Quiroga Teixeiro, and S. K. Turitsyn, Pure Appl. Opt. **4,** 281 (1995); see also Yu. S. Kivshar and S. K. Turitsyn, Phys. Rev. E **49,** 2536 (1994).
11. R. Ohhira, A. Hasegawa, and Y. Kodama, Opt. Lett. **20,** 701 (1995).