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Long scale evolution of a nonlinear stochastic dynamic system for modeling market price bubbles

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Abstract

This Letter investigates the stochastic dynamics of a simplified agent-based microscopic model describing stock market evolution. Our mathematical model includes a stochastic market and a sealed-bid double auction. The dynamics of the model are determined by the game of two types of traders: (i) 'intelligent' traders whose strategy is based on nonlinear technical data analysis[1] and (ii) 'random' traders that act without a consistent strategy. We demonstrate the effect of time-scale separations on the market dynamics. We study the characteristics of the market relaxation in response to perturbations caused by large cash flows generated between these two groups of traders. We also demonstrate that our model exhibits the formation of a price bubble[2] and the subsequent transition to a bear market[3].

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1. Introduction

Mathematical modeling of the complex evolution of the stock market has recently been the focus of intensive research [1–4]. Widely accepted mathematical approaches to market modeling can be divided into two main groups. The first group includes modeling based on traditional statistical analysis of high frequency financial market time series [2,3,5]. Research which utilized this approach originated in the early 1960s [6,7]. The second group of mathematical

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[1] Technical analysis – a study of stock market price evolution, trading volume, trends, and patterns based on historical market data. The goal of technical analysis is to develop successful trading strategies.

[2] Stock market price bubble – a steady increase of a stock's price over a long time period following by a sharp transition to a price declining phase.

[3] Bear market – a macroscopically long stage of a market evolution when the stock price declines significantly, 15% or more.

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approaches considers the market as a dynamic system. Among these models, agent based computer modeling of the financial markets [8–15] is of considerable importance. In this approach the elementary building blocks of the market are the agents which can buy and sell assets based on complex intelligent decisions. Market decisions are based on the agent's personal strategy, learning ability, availability of information and many other factors.

An agent based model of the markets can potentially capture the underlying features of the actual stock market which consists of human agents. Certainly, human involvement makes the system much more complicated than classical stochastic systems such as those studied by physicists in thermodynamics. It has been shown that the behavior of the stock market is non-equilibrium, or has dynamic equilibria, and involves highly non-stationary processes [1,3,16].

The investigation of real financial markets is extremely complicated. For example, in order to prove their findings, researchers cannot repeat the 'experiment' many times under controlled conditions. Computer simulations of microscopic market models allow for the repetitive investigation of specific phenomenon while letting researchers vary and control the parameters of the system. If some interesting feature appears, one can trace the evolution of the phenomena over all time scales. This makes agent-based computer simulations a powerful tool that allows for developing new models and testing already existing phenomenological economic models.

Recently, the amount of financial information available for investigation has been growing in an explosive way. Stock market historical data is collected at the rate of hundreds of megabytes per day. With the continuous expansion of electronic exchanges and their trading hours, the amount of historical data increases even faster. This information opens a broad range of opportunities for model verification, for understanding of the basic mechanisms that determine the dynamics of the market and for developing new winning strategies.

Despite increasing research, this area is still new and many questions about market and agent design remain unanswered. Current efforts are focused on two types of problems: (i) modeling of artificial (virtual) markets which mimic as closely as possible actual markets [8,11–15], and (ii) designing prototypes for the actual markets which are more efficient than the current ones [17].

Our simplified mathematical model describes an artificial market with only two distinct types of traders. The first group of traders randomly submit their orders. Their role is to keep the market constantly running. Their cash and share inventories reflect the corresponding values affected by the exchange. The second group of 'intelligent' TA traders (those who make their decisions on the basis of Technical Analysis [18]) represent the 'winners' who 'pump' money out of the market. The decision making strategy of the TA traders in our modeling is based on the Bollinger Bands approach [18].

Our model exhibits the formation of a price bubble and the subsequent transitions to a bear market when the stock price declines virtually to zero. The microscopic simulation we present below clarifies the underlying factors that determine the dynamics of the stock price. The TA traders rarely participate in the market, but their 'intelligent' actions influence the re-distribution of money and shares among the two groups. The random traders determine the 'fast' stock price dynamics. It appears that on a macroscopic time scale the stock price reflects the ratio of cash and share inventories owned by this group of random traders. On a mesoscopic time scale, we find that the market has a finite response time to the TA traders' injection of money into the market.

The Letter is organized as follows: In Section 2 we define our model. In Section 3 we present the results of numerical simulations of the model. Section 4 concludes the current article. The Appendix describes the Bolliger Bands technical indicator used by the group of 'intelligent' computer agents.

2. Model description

We will consider an artificial simplified market model, which operates with only one tradeable stock. The time-dependent price of this stock is denoted by $P(t)$. There are two distinct groups of market investors who trade this equity. The first group consists of random traders. The number of random traders participating in the market is N_{RND}. Random traders do not have any consistent strategy. Rather, they make random guesses concerning future stock price and their action (buy, sell, or wait) is random also. Similar random traders were first introduced into the asset pricing literature by De Long et al. [19].

The second group of investors is composed of 'intelligent' traders. The number of the TA traders is N_{TA}. They investigate the history of the asset price and form their decisions based on a consistent strategy, which they apply to the historical data. They believe that the asset has a fundamental, or fair, value and the evolution of the real price is a stochastic walk around that value. This mind-set allows them to consider the deviation of the price from its fundamental value as arbitrage opportunities from which they may profit after a return of the price to the underlying fair value. The 'intelligent' traders utilize technical analysis indicators [18] to evaluate this deviation.

The market evolves in discrete time, t. At any time step t every trader has a cash amount $C_i(t)$ (real) and number of shares $S_i(t)$ (integer). Each trader knows the value of the current price $P(t)$ of the asset and the price history at all previous time steps. Based on this information all traders make a prognosis $\tilde{P}(t+1)$ of the asset's price at the next time step. After the prognoses are made, the traders submit their orders to the exchange, if they decide to do so. The auctioneer collects all orders and calculates the new price for the asset. She then executes the orders that agree with the new price.

2.1. Make prognosis and submit orders

At each time step t, before submitting an order, every agent makes a personal prognosis $\tilde{P}_i(t+1)$ about the price of the stock at time $t+1$. The prognosis is based only on the historical information available at time t.

The traders, whether TA or random, do not use any distinct strategy to decide on the value of the new price of the asset. Their guess about the new price comes from a large number of unpredictable sources. The resulting prediction simply assumes that the new price is the current one shifted by a random number

$$\tilde{P}_{t+1} = P_t(1 + \sigma r), \qquad (1)$$

where r is an independent, identically distributed random number ($r \in [-1, +1]$) and σ is a variance of the stochastic component ($\sigma = 0.05$ in current simulations).

Next, the traders decide if they want to submit an order. At this step the behavior of the traders belonging to the distinct groups differs. The random traders do not have a specific personal strategy and at each time step their decision is under the strong influence of external sources of meaningless information. As a result, the random traders decide whether to buy or to sell the asset, or to keep their current position with an equal probability of $1/3$. If the ith random trader decides to buy an asset he places a buy order with $\tilde{P}_i(t+1)$ as the limit price. Otherwise, if she decides to sell the asset, a limit sell order is submitted by the trader with the limit price also being $\tilde{P}_i(t+1)$. Finally, if the given trader chose to keep her position, no further action on her part takes place.

At the same time, the group of 'smart' TA traders uses technical analysis to make an intelligent decision on whether to buy, to sell, or to wait until the next time step. In the current simulation these traders use the 'Bollinger Bands' (see Appendix) indicator to generate their buy/sell signals. This indicator involves two slowly varying bands from the lower and upper sides of the quickly moving curve of the stock price. These bands create an envelope around the stochastically changing price graph. The parameters for the Bollinger Bands used in this Letter are as follows (see Appendix for details). The period used to calculate the exponential moving average and the price deviation is 20 time steps. The upper and lower bands are at a distance of 3 standard deviations from the moving average value. In the notation of the Appendix we can write: $T_{\mathrm{EMA}} = 20$, and $C_{\mathrm{Upper}} = C_{\mathrm{Lower}} = 3$.

The ith TA trader first makes a prognosis about the next price according to the stochastic formula, presented above, Eq. (1). If the prognosis appears to be lower than the lower band, $\tilde{P}_i(t+1) < B_{\mathrm{Lower}}(t)$, the trader considers the security oversold and places a buy order with the limit price $\tilde{P}_i(t+1)$. In the opposite case when the prognosis jumps over the upper band, $\tilde{P}_i(t+1) > B_{\mathrm{Upper}}(t)$, the trader expects the price to drop and she places a sell order with the limiting price $\tilde{P}_i(t+1)$. In the case when the prognosis shows the price to be in between the Bollinger

Bands, $B_{\text{Lower}}(t) \le \tilde{P}_i(t+1) \le B_{\text{Upper}}(t)$, the trader keeps his current position unchanged.

When the ith trader decides to submit a buy order she is going to use all available cash to buy $int[C_i(t)/\tilde{P}_i(t+1)]$ units of stock. If she is going to submit a sell order, he wants to convert all available securities, $S_i(t)$, into cash, $S_i(t)\tilde{P}(t+1)$.

2.2. Calculate new price

After all investors have placed their limit orders the auctioner uses a sealed-bid double auction to calculate the new price for the asset, $P(t+1)$. The auctioner collects two lists: 1) a list of bids to buy long with the amounts of shares desired, and 2) a list of offers to sell with the amount of shares offered.

Two curves are then calculated: the demand vs. price and the supply vs. price. Their intersection determines the new price for the trade, $P(t+1)$. Because the supply and demand curves have a stepwise shape it is possible that they may intersect over a finite price region. If this happens the right most value of the common region is accepted as the new price for the trade. The volume of the trade executed at price $P(t+1)$ will be $V(P) = \min[\text{Supply}(P), \text{Demand}(P)]$. By implementing the above described algorithm for finding the new price the auctioner maximizes the volume of shares participating in the transactions. In some auctions a slightly different algorithm is implemented in which the auctioner intends to maximize the amount of cash that changes hands during the trade. It was shown in Refs. [12–14] that both of these algorithms produce almost identical results.

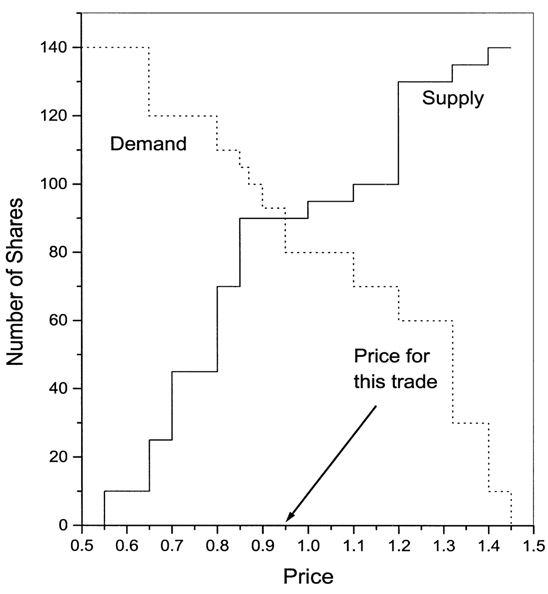

Fig. 1. Example of supply and demand curves vs. price. The intersection of these curves determines the new price for the trade.

2.3. Execution of orders

Next, the auctioner executes sell orders with limit prices not lower than the new price, $P(t+1)$, and buy orders with limit prices not greater than this price. We match orders by considering first sellers with the lowest price and buyers with the highest one.

3. Simulation

The simulation proceeds in discrete time steps. The computer program performs the following actions during each time step:

1. all traders make their personal prognoses about the next price of the security;
2. based on the prognosis made and on their personal strategy, every trader makes a decision to either: (a) place a limit sell order, (b) place a limit buy order, or (c) keep the current position unchanged and do not to participate in the upcoming trade;
3. the auctioner collects all submitted limit orders and calculates the new stock price for the trade;
4. the auctioner executes transactions by matching buy and sell orders in agreement with the trading price;
5. the auctioner informs all traders about the new price and updates the number of shares and cash for the traders whose orders participated in the transactions.

In the current Letter we investigate a market consisting of 300 random traders and 300 TA traders working with Bollinger Bands. Initially, every trader has 1000 shares of stock and 1000 units of money.

The group of random traders participates in the market starting at $t = 0$. The TA traders start trading only at $t = 500$ since the calculation of the Bollinger Bands indicator requires historical data.

3.1. Price and volume

The evolution of the stock price is shown in Fig. 2. On a large time scale the picture clearly shows the two evolutionary modes of the system: the initial bull market, and the eventual bear market that leads to a price that approaches zero. On a smaller scale, the price moves in rather stable up and down trends over large numbers of time steps.

For quantitative investigation of the price bubble we performed a comparative analysis of the market evolution characterized by a different normalized number of random traders $\tilde{N}_{\mathrm{RND}} = N_{\mathrm{RND}}/(N_{\mathrm{TA}} + N_{\mathrm{RND}})$. Total number of traders are fixed in each case $N_{\mathrm{TA}} + N_{\mathrm{RND}} = \mathrm{Const} = 600$, therefore total number of shares and cash is the same in every virtual market. We simulate market dynamics for the following set of parameter values $\tilde{N}_{\mathrm{RND}}$, $\tilde{N}_{\mathrm{RND}} =$ 0.25,0.3,0.333,0.417,0.5,0.583,0.667,1. The maximum value of the share price for each scenario P_{peak} as a function of $\tilde{N}_{\mathrm{RND}}$ is presented on Fig. 3. If all of the traders are random traders, $\tilde{N}_{\mathrm{RND}} = 1$, the market

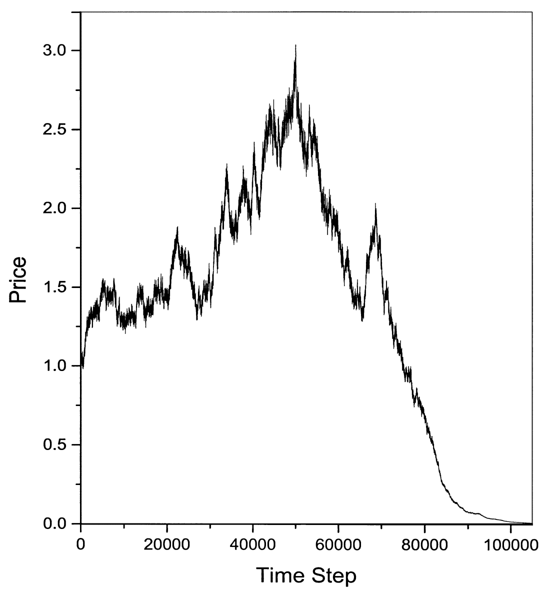

Fig. 2. The evolution of the stock price vs. time.

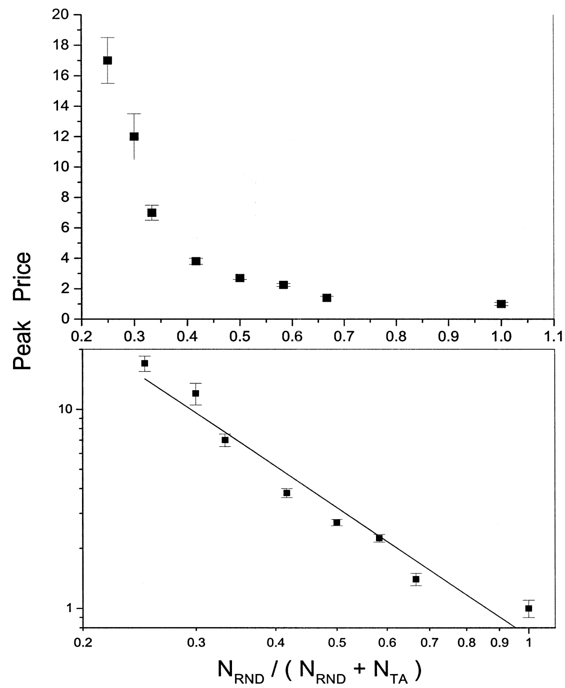

Fig. 3. The maximum value of the stock price, P_{peak}, vs. normalized number of random traders, $\tilde{N}_{\mathrm{RND}} = N_{\mathrm{RND}}/(N_{\mathrm{TA}} + N_{\mathrm{RND}})$. The lower panel shows the graph in logarithmic scales. It follows that $P_{\mathrm{peak}} \sim \tilde{N}_{\mathrm{RND}}^{-2.15}$.

is in an equilibrium state. The share price in this case fluctuates around its equilibrium value $P_{\mathrm{eq}} = P_{\mathrm{peak}} = 1$. The market reacts to an increase of the fraction of intelligent traders with an increased P_{peak}. The function $P_{\mathrm{peak}} = P_{\mathrm{peak}}(\tilde{N}_{\mathrm{RND}})$ is monotonically decreasing on the interval $\tilde{N}_{\mathrm{RND}} \in [0.25,1]$. The function $P_{\mathrm{peak}} = P_{\mathrm{peak}}(\tilde{N}_{\mathrm{RND}})$ in both logarithmic scales is shown on the lower part of Fig. 3. It follows from this graph that the peak price decays as $P_{\mathrm{peak}} \sim \tilde{N}_{\mathrm{RND}}^{-2.15}$.

The trading volume graph is presented in the upper panel of Fig. 4. Comparing this with the previous figure, one can see that trading volume slows down and reaches its minimum approximately at the time of the stock price absolute maximum, $t \approx 50\,000$.

The lower panel in Fig. 4 shows the intergroup trading volume, i.e., the change in the number of shares owned by one of the two groups during a trade. The insert shows a magnification of a short time interval $50\,000 < t < 51\,000$. It is evident that the intergroup trade is a ‘rare event’ because one can

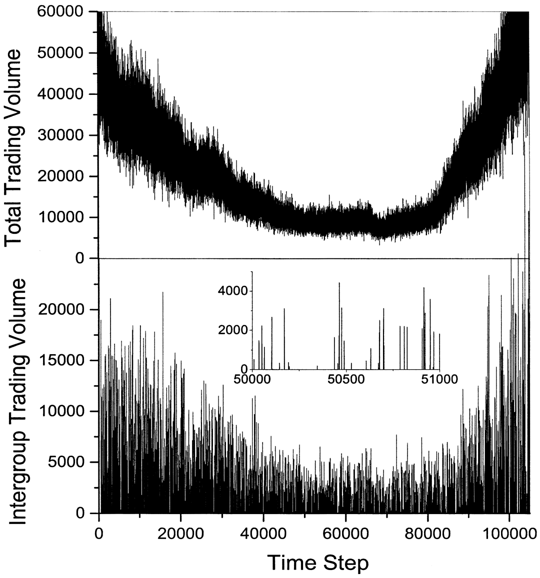

Fig. 4. The evolution of total trading volume (the upper panel) and intergroup trading volume (the lower panel) vs. time. The insert at the lower panel magnifies a short time region ($50\,000 < t < 51\,000$) of the graph which clearly shows that the intergroup trades are 'rare events'.

find only about 30 intergroup trades during the thousand time steps shown in the insert. The black shaded region in the lower panel means that the full graph in that panel consists of sharp spikes between zero and positive values. Such spikes are distributed sparsely over the time axis. We can conclude that, most of the time, trades occur between members of the same group and that the average intergroup trading volume always stays much lower than the total rading volume.

3.2. Adiabatic approximation. Equilibrium price

Now we are in a position to assume that the adiabaticity is present in our system. We can distinguish two subsystems in the market under investigation: (i) the 'fast' subsystem consists of the group of random traders, and (ii) the 'slow' subsystem is represented by the TA traders. The random traders always participate in the trade. On the other hand, as it follows from the picture of the sparse intergroup trades (see insert in Fig. 4), the 'smart' TA traders rarely participate in the market and change their positions.

Consider separately the dynamics of the fast subsystem only and assume that there are only random traders on the market. These traders randomly offer buy and sell orders to the auctioner and they always use all owned cash or shares in their orders. This system has a time-independent equilibrium price for the stock. The traders use their total cash or shares in their orders, therefore the equilibrium stock price is proportional to the ratio of the total money to the total number of shares on the market. Hence, the equilibrium price, $P_{\mathrm{eq}}(t)$, is given by the following expression:

$$P_{\mathrm{eq}}(t) = \frac{\sum_{\mathrm{RND}} C_j(t)}{\sum_{\mathrm{RND}} S_j(t)}, \tag{2}$$

where both summations are taken for random traders only.

Note that if the initial ratio of cash and shares owned by the random traders persists throughout the lifetime of the market the equilibrium stock price is equal to 1 (see the description of the initial conditions presented above). The deviation of the equilibrium stock price from its initial value reflects the slow re-distribution of cash and shares among the different groups of traders.

The evolution of the market can be viewed as follows. The random traders constantly participate in the exchange of money and shares always utilizing the total inventory they own. The equilibrium price of the stock is equal to the ratio of the money and the number of shares which are belong to the group of random traders. The equilibrium price changes rarely over time. This happens only when the Bollinger Bands technical indicator signals to the TA traders to change their position, i.e., to buy or sell the stock. If the distribution of the buy and sell orders (see Fig. 1) is favorable and the TA traders are able to participate in the transaction, an intergroup trade occurs and the number of shares and the amount of money belonging to the group of random traders will change. As a result, the time dependent equilibrium stock price will change.

Fig. 5 shows the time evolution of the equilibrium price. It is evident from this graph that the equilibrium stock price behaves very similar to the actual price of the stock (see Fig. 2). One can also see that

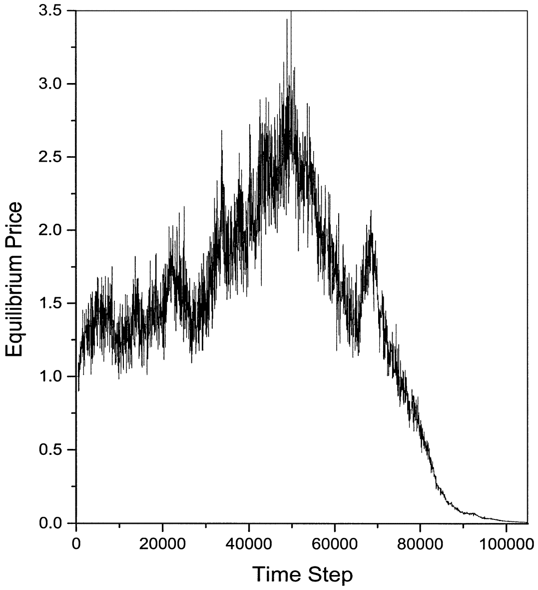

Fig. 5. The evolution of the equilibrium stock price vs. time. The equilibrium stock price, Eq. (2), is calculated under the assumption that trading occurs primarily between the random traders, i.e., within the fast subsystem.

the deviation of the equilibrium price from its average value is greater than the deviation of the actual stock price. This gives us a hint that the actual stock price always follows, and attempts to catch up to, the stochastically oscillating equilibrium price.

The next figure, Fig. 6, shows the number of shares and the cash inventory for an average trader in each group. To calculate the values shown in the figure at every time step we take the sum of the corresponding value over all traders in the group and divide it by the number of traders in the group (300 for every group in the current simulations).

We can see that as time proceeds the ‘smart’ TA traders continuously increase their cash inventory. Their simple and well-known strategy – buy low and sell high – works, and, after completing the cycle cash-shares-cash, they become richer. It is interesting to note that they manage to increase their amount of money during both bull and bear markets. During the bull market, when the average stock price slowly goes up, the Bollinger Bands strategy brings a profit because the sell signal is generally generated at a higher price than the buy signal was triggered. During the period of the bear market it appears that the stock price decreases at a slow enough rate so that the sell signals still occur at a higher price relative to the price of the last buy signal.

Another fact we can derive from Fig. 6 is that the stock price hits its high at a time when the random traders have the highest ratio of cash to owned shares (the time region near $t = 50\,000$). This observation agrees with the adiabatic picture described above. Another prediction of the adiabatic framework is that during the bull market the random traders should have less shares than cash, while during the bear market the opposite relation of cash and shares must be observed. Again, we can conclude from Fig. 6 that these two facts indeed take place in the simulations.

The behavior of the ‘cash’ and ‘shares’ curves in Fig. 6 allows one to explain the transition of the market from the bull stage to the bear one. During the bull market the TA traders prefer to have more shares than cash. The difference between the two commodities is approximately constant ($5000 < t <$

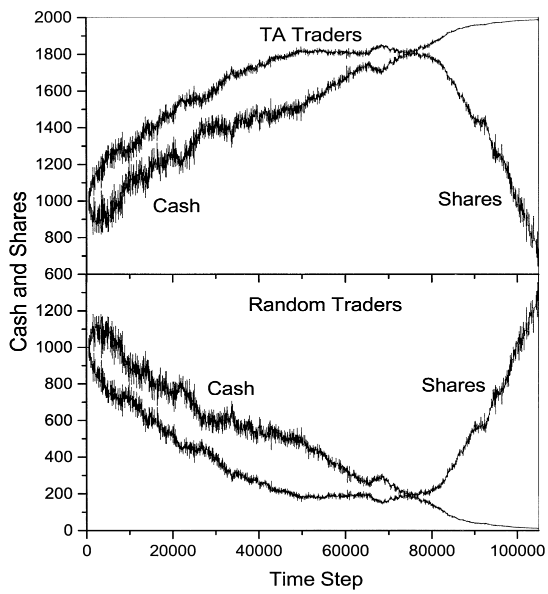

Fig. 6. The evolution of the number of shares and the cash inventory for an average trader in each group vs. time. The upper panel presents the data for the group of TA traders, and the lower panel presents the data for the random traders. The values shown are ensemble averages, i.e., the sum of a corresponding value taken over all traders in the group is divided by the total number of traders in this group.

30 000) and even increases ($30\,000 < t < 50\,000$). The difference between the cash and the share inventories of the random traders behaves analogously. This situation can not hold forever because the total number of shares on the market is limited and the 'shares' curve of the upper panel in Fig. 6 reaches saturation while the 'cash' curve still rises linearly. These two curves start approaching each other at $t \approx 50\,000$ and the bear market results.

An average trader's wealth is presented in Fig. 7. The wealth of the ith trader is given by $W_i(t) = C_i(t) + P(t)S_i(t)$. The average value shown in the figure is calculated in the same manner as the cash and share average inventories, shown in Fig. 6. As one can see in the lower panel of the Fig. 7, the wealth of the random traders consistently decreases with time. It appears that the strong stock price variation during the price bubble does not exhibit much influence over their wealth. During the bull market stage the stock inventory of random traders is significantly lower than their cash inventory, so the stock price has little effect. During the bear market stage the stock inventory of the random traders increases significantly, but the stock price decreases fast enough to not produce any visible effects on the wealth. Conversely, the wealth of the TA traders strongly depends on the direction of the market trend. It follows the stock price during the price bubble period (compare with Fig. 2). When the asset's price diminishes virtually to zero at the final stages of the bear market, the wealth of the traders decreases to its cash inventory value. In the end, the random traders lose all their money and acquire valueless assets while the 'smart' TA traders convert all their shares to cash.

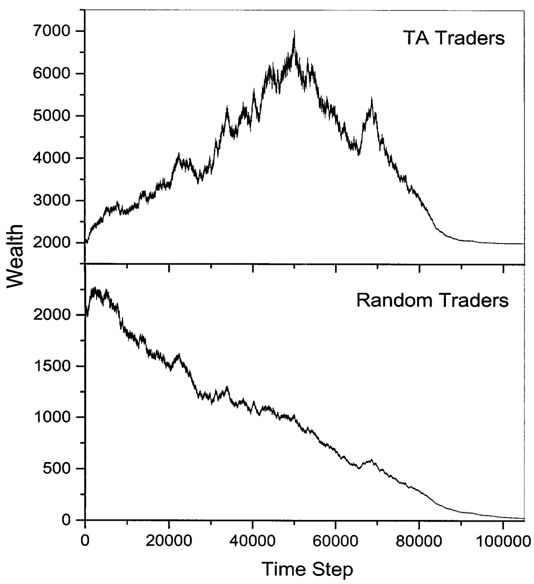

Fig. 7. The evolution of the wealth for an average trader in each group vs. time. The upper panel presents the data for the group of TA traders, and the lower panel presents the data for the random traders. Each value is averaged over an ensemble of traders, i.e., the total wealth of all traders in the group is divided by the total number of traders in this group.

3.3. Market reaction to large intergroup transactions

We now concentrate our attention on a more detailed picture of the stock price evolution. Namely, we investigate the price behavior on a mesoscopic scale on the order of a hundred time steps or less. This scale reveals how the price of the stock reacts to the intergroup transactions, which occur when the TA traders enter or exit the market.

Fig. 8 shows details of the stock price evolution during a short time period of the bull market, $66000 < t < 66500$. In addition to the price of the security, $P(t)$, and the two Bollinger Bands the figure also shows the equilibrium price of the stock, $P_{eq}(t)$. We can see that the price fluctuates between the Bollinger Bands, bouncing back when it touches either the lower or the upper band. The dashed line represents the equilibrium price which changes every time money transfers occur between the two groups. The price curve tends to follow the equilibrium price curve, at least the former tends to stay not too far from the latter. The stock price reverses its walk especially sharply when a large intergroup transaction happens. This, for example, is the case at $t = 66092$ when the TA traders entered the market and bought 6381 shares of stock. The equilibrium price jumped from 1.397 to 1.756. The stock price bounced back from the lower to the upper Bollinger Band in 7 time steps, from $t = 66092$ to $t = 66099$. The 'smart' TA traders then began selling their asset considering it overvalued, i.e., far above its fair value. The selling process by TA traders extends for a few dozen time steps until the equilibrium price lowers

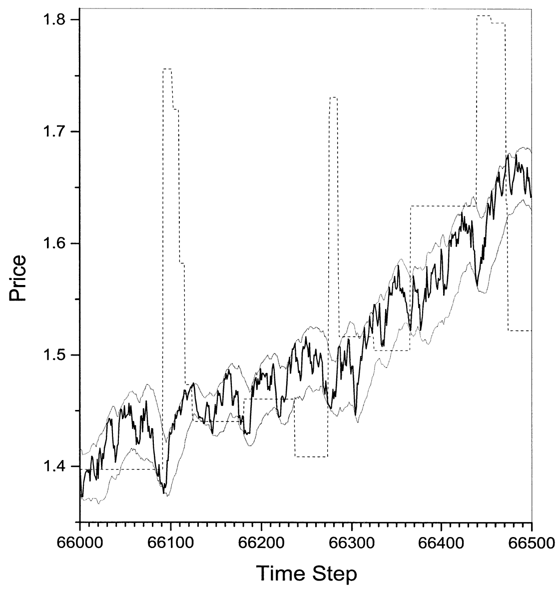

Fig. 8. The stock price, the Bollinger Bands, and the equilibrium price during the bull market. The stock price is the central stochastically oscillating curve surrounded by the lower and the upper Bollinger Bands. The equilibrium price is shown by a dashed line.

down to the actual stock price. A few similar situations of strong jumps in equilibrium price and follow up bounces of the actual stock price can be found on this figure.

Fig. 9 shows details of the stock price evolution during a short time period of the bear market, 69600 $< t < 70200$. Again, the price of the security, $P(t)$, two Bollinger Bands, and the equilibrium price of the stock, $P_{eq}(t)$, are shown in the figure. On this figure one can also notice a sequence of large intergroup transactions which change the equilibrium price sharply, for instance, at $t \approx 69740$, and $t \approx 69940$. The response of the market to such large intergroup transactions is similar to that discovered on the previous figure – the stock price walks stochastically toward the new equilibrium price.

As we noticed from the two previous figures, Figs. 8 and 9, the price movement on a mesoscopic time scale exhibits stable patterns with up and down trends. A possible explanation for the existence of such self-supported patterns of price time evolution is that the market has a finite characteristic time of reaction to a sharp change in the equilibrium price. In other words, when a large intergroup transaction takes place, the money (or shares) instantaneously injected into the group of random traders is initially obtained only by a fraction of traders in the group. It takes some time for the created imbalance to spread uniformly over all traders in the group.

Thus, one can roughly draw the following scenario of the stock price evolution shown in Fig. 10. The figure schematically represents two situations in which the stock price evolves in up and down trends. Intergroup transactions occur when the price graph touches the Bollinger Bands. The ‘smart’ TA traders buy the shares when the price drops below the lower band and they sell their owned stock when its price reaches the upper band. Immediately after the large intergroup transaction the price graph changes its direction but its slope forms the same angle with the horizontal axis after each such transaction (the value of this angle is determined by the characteristic response time for the system). One can easily see that when the stock price has an up-trend the time period during which TA traders keep shares is longer than the one during which they own cash. It follows

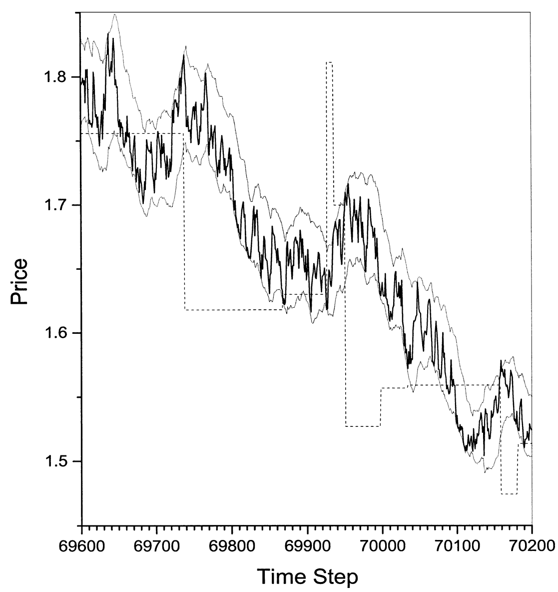

Fig. 9. The stock price, the Bollinger Bands, and the equilibrium price during the bear market. The stock price is the central stochastically oscillating curve surrounded by the lower and the upper Bollinger Bands. The equilibrium price is shown by a dashed line.

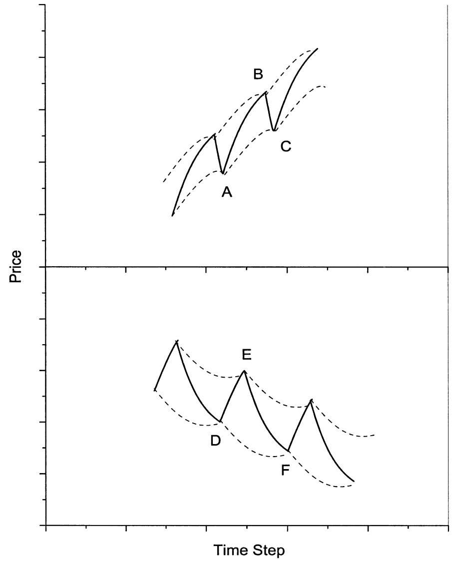

Fig. 10. The schematic representation of the stock price evolution. The market exhibits a finite response time after a large intergroup transaction takes place. The solid curve shows the price while the two dashed curves correspond to the Bollinger Bands. The upper panel represents the self-supported upward price evolution when the TA traders keep shares longer than cash. The lower panel shows the downward price evolution scenario when the TA traders, on average, keep money longer than shares.

from this fact that the time stretch between points A and B is longer than the one between between B and C. This makes the scenario consistent with the situation shown in Fig. 6, i.e. there is a lack of shares for the random traders when the stock price moves in an up-trend. When the stock price moves in a down-trend the situation is opposite and the TA traders keep cash longer than shares (time difference between D and E is larger than between E and F). Again this picture agrees with the simulation results presented in Fig. 6.

3.4. Market characteristic response time

The scenario described above is based on the assumption that the market at hand is characterized by a finite response time to an intergroup transaction that changes the amount of money owned by the random traders. In order to verify this assumption we conduct the following computer experiment. We launch the market with 300 random traders where initially each one owns 1000 shares and 1000 units of money. At time step $t = 1000$ of the market evolution we inject an additional 10% of money. This is done by adding additional 1000 units of money instantaneously to the first 30 traders. We then allow the market to evolve further while the history of the stock price is recorded. We repeat the simulation 500 times, each time shifting the money injection moment by one time step forward. The average over 500 realizations of the stock price increase following the money injection moment is calculated. Finally, we calculate the same average of the stock price change without the money injection. The latter is subtracted from the former:

$$\Delta(t) = \langle P(t) \rangle_{\text{Money injection}} - \langle P(t) \rangle . \qquad (3)$$

Fig. 11 represents the market response to the instantaneous injection of money. The injected cash increases the equilibrium price from $P_{eq} = 1$ to $P_{eq} = 1.1$ (dashed line in Fig. 11). Indeed, the market response is not instant and the stock price approaches exponentially the new equilibrium price.

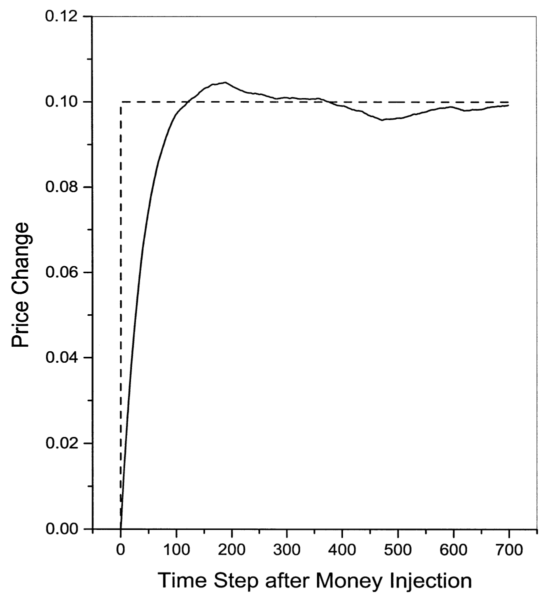

Fig. 11. The increase in the stock price after the money injection. The solid curve shows the stock price after the money injection minus the stock price without the injection. The dashed line is the change in the equilibrium price for the stock. The result is averaged over 500 runs.

The characteristic response time of this market is $\tau \approx 40$. It seems that one can even observe the residual oscillations of the stock price around its equilibrium value.

3.5. Distribution of returns

We now look at the statistical properties of the market under investigation. The question we would like to address is: does the presence of 'smart winners', TA traders, change the probability distribution of the stock price fluctuation? We compare probability distributions of relative price changes (returns) for two markets, with and without TA traders. The return, $G(t)$, is calculated according to the following equation:

$$G(t) = \frac{P(t+1) - P(t)}{P(t)}. \qquad (4)$$

The upper panel of Fig. 12 presents the histogram of returns for the market of 300 TA traders and 300 random traders discussed throughout the Letter. The lower panel of this figure gives the histogram of the returns for the market consisting of only random traders. The former market has the group of 'smart' TA traders who, on average, 'buy low and sell high', i.e., they rarely, but precisely, enter and exit the market making money during these cycles. The latter market is homogeneous and does not have 'smart' traders. The stock price always stochastically oscillates around $P_{eq} = 1$ and there is no steady money flow from losers to winners. Visually, we cannot see much meaningful difference between the two histograms presented in the figure. This is not a surprise because the intergroup transactions in our market are rare events. Remember, that according to the results shown in Fig. 4 only one such transaction occurs per 30 time steps.

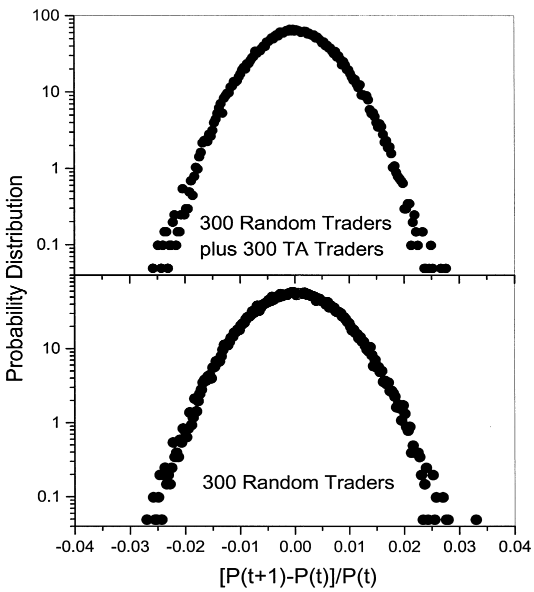

Fig. 12. The probability distribution of the returns. The upper panel gives the distribution of returns for a market with 300 TA traders and 300 random traders, while the lower panel shows the distribution for one consisting of 300 random traders only.

4. Summary

The simulations presented in this Letter correspond to a very simple artificial stock market. The market trades only one security (one type of stock), the traders can submit only limit orders (a limit order corresponds to an order by a trader to buy/sell a stock at a price, which is limited from above/below respectively). There are only two types of traders – random traders and traders with the Bollinger Bands strategy. Despite these very strong limitations the behavior of this market reveals features of real financial markets. This market exhibits macroscopically long stages which can be described as a bull market or a bear market. The transitions between the bull and bear markets always exhibit price bubble behavior. In our simulations the stock price increased by about 3 times over its original value during the bull market stage. Then the stock price collapsed and the market evolution entered the bear market stage.

We investigated the dependence of the peak value of the price bubble versus a relative number of random traders acting in the market. The total number of traders was equal to 600. The peak price is decreasing at the interval $\tilde{N}_{RND} \in [0.25,1]$ as $P_{peak} \sim \tilde{N}_{RND}^{-2.15}$.

The above behavior of the stock price can be explained within the framework of adiabatic dynamics, which is proposed in this Letter. We distinguish two groups of traders: the 'fast' one and the 'slow' one. The group of random traders represent the 'fast' traders. They are consistently trading and the ratio of their inventories determine the evolution of the stock price. It is worth mentioning that the stock price stochastically oscillates around the time-dependent equilibrium price, which is given by the ratio of cash and share inventories owned by the random traders, Eq. (2).

During the bull market stage the 'intelligent' TA traders on average keep increasing share position and decreasing their cash position. This supports the market's upward trend, Fig. 6. During the bear market the random traders are left with an increasing share position and decreasing cash position, again, supporting the market's downward move. The stock price eventually goes to zero at the final stage of the market evolution because the 'intelligent' TA traders keep all the available money which leaves the random traders with priceless shares.

On a mesoscopic time scale the stock price dynamics can be described as a sequence of stochastic walks toward the jumping with a large amplitude equilibrium price, Figs. 8, 9. After the TA traders enter the market by injecting a large amount of money or shares, the equilibrium price changes sharply. The reaction of the stock price to these injections causes the finite response time of the market, Fig. 8.

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Appendix A. Exponential moving average and Bollinger bands

A.1. Exponential moving average

A 'moving average' is an indicator that shows the average value of a security's price over a period of time [18]. The time period is chosen to be the end of the historical data. For instance, the average is calculated over a month with today as the last day. There are five popular types of moving averages: simple (also referred to as arithmetic), exponential, triangular, variable, and weighted.

An exponentially weighted or 'exponential moving average' (EMA) is calculated by adding a portion of today's price to yesterday's moving average value. EMA places more weight on recent prices.

$$\mathrm{EMA}(t+1) = C_{\mathrm{EMA}} P(t) + (1 - C_{\mathrm{EMA}})\mathrm{EMA}(t), \tag{A.1}$$

where $C_{\mathrm{EMA}} = 2/(T_{\mathrm{EMA}} + 1)$, and T_{EMA} is the time period for the average.

A.2. Bollinger bands

Bollinger Bands are a particular type of trading bands used by technically based investors. Trading bands are lines plotted around the fluctuating price graph to form an envelope which contains a majority of the data. Bollinger Bands are two lines plotted around the moving average at a distance which

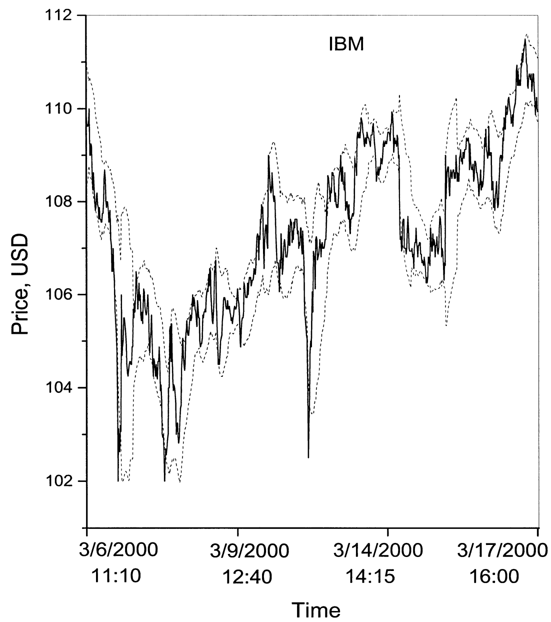

Fig. 13. Example of Bollinger Bands drawn around the stock price of International Business Machines Co. (IBM). The stock price is shown at 5 minute time steps for the time period between 3/6/2000 11:20 and 3/17/2000 16:00. (NYSE intraday trading hours extend from 9:30 to 16:00). For the calculation of the Bollinger Bands we use the following parameters: $T_{\mathrm{EMA}} = 20$ and $C_{\mathrm{Upper}} = C_{\mathrm{Lower}} = 3$.

varies with price volatility [18]. Such bands are self-adjusting: they widen during volatile markets and contract during calmer periods. This technical indicator was invented by John Bollinger.

In this Letter, to construct Bollinger Bands, we use the exponentially weighted moving average, Eq. (A.1), over the period of 20 time steps, $T_{EMA} = 20$ as a middle band. The standard deviation of a price return, given by the following equation, over the same period is used as a measure of the price volatility:

$$\Delta(t) = \sqrt{\frac{\sum_{k=0}^{T_{EMA}-1} \left[P(t-k) - P(t-k-1) \right]^2}{T_{EMA}}}, \tag{A.2}$$

where $\Delta(t)$ is the standard deviation of the price return over the T_{EMA} steps of historical price data, $P(t)$.

The upper and lower bands are then plotted around the middle band, $\mathrm{EMA}(t)$, according to the following equation:

$$B_{Upper}(t) = \mathrm{EMA}(t) + C_{Upper}\Delta(t),$$
$$B_{Lower}(t) = \mathrm{EMA}(t) - C_{Lower}\Delta(t), \tag{A.3}$$

where C_{Upper} and C_{Lower} are coefficients on the order of 1. For the calculations, presented in this Letter we use the following values for these coefficients: $C_{Upper} = C_{Lower} = 3$.

The basic interpretation of Bollinger Bands is that prices tend to stay within the upper- and lower-band. The ‘buy signal’ is generated when the stock price goes below the lower band, while the ‘sell signal’ appears when the price reaches the upper band. The distinctive characteristic of Bollinger Bands is that the spacing between the bands varies based on the volatility of the prices. During periods of extreme price changes (i.e., high volatility), the bands widen to become more forgiving. During periods of stagnant pricing (i.e., low volatility), the bands narrow to contain prices. (See Fig. 13.)

References

[1] P.W. Anderson, J.K. Arrow, D. Pines, The Economy as an Evolving Complex System, Addison-Wesley, Redwood City, CA, 1988.
[2] J.-Ph. Bouchaud, M. Potters, Theory of Financial Risk: From Statistical Physics to Risk Management, Cambridge Univ. Press, Cambridge, UK, 1999.
[3] R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge Univ. Press, Cambridge, UK, 1999.
[4] K.N. Ilinksi, Physics of Finance. Gauge Modelling in Nonequilibrium Pricing, Wiley, New York, 2000 (in press).
[5] R.N. Mantegna, H.E. Stanley, Nature 376 (1995) 46.
[6] B. Mandelbrot, J. Bus. 36 (1963) 394.
[7] E.F. Fama, J. Bus. 38 (1965) 34.
[8] W.B. Arthur et al., Asset Pricing under Endogeneous Expectations in an Artificial Stock Market, in The Economy as an Evolving, Complex System II, Addison-Wesley, Reading, Mass., 1997.
[9] T. Lux, Econ. J. 105 (1995) 881.
[10] M. Levy, H. Levy, S. Solomon, J. Phys. I 5 (1995) 1087.
[11] P. Bak, M. Paczuski, M. Shubik, Physica A 246 (1997) 430.
[12] K. Steiglitz, M.L. Honig, L.M. Cohen, A Computational Market Model Based on Individual Action, Chapter 1, in: Scott Clearwater (Ed.), Market-Based Control: A Paradigm for Distributed Resource Allocation, World Scientific, Hong Kong, 1996.
[13] K. Steiglitz, D. Shapiro, Comput. Econ. 12 (1998) 35.
[14] H. Mizuta, K. Steiglitz, E. Lirov, 4th Workshop on Economics with Heterogenous Interacting Agents, Genoa, June 4-5, 1999.
[15] C. Busshaus, H. Rieger, Physica A 267 (1999) 443.
[16] J.D. Farmer, preprint adap-org/9812005.
[17] Hearing on Financial Marketplace of the Future http://www.senate.gov/~banking/00_02hrg/022900/
[18] S.B. Achelis, Technical Analysis, From A to Z, Irwin, 1995.
[19] J.B. DeLong, A. Shleifer, L.H. Summers, R.J. Waldman, J. Polit. Econ. 98 (1990) 703.