Inelastic interchannel collisions of pulses in optical fibers in the presence of third-order dispersion

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We study the effect of third-order dispersion on the interaction between two solitons from different frequency channels in an optical fiber. The interaction may be viewed as an inelastic collision in which energy is lost to continuous radiation owing to nonzero third-order dispersion. We develop a perturbation theory with two small parameters: the third-order dispersion coefficient d_3 and the reciprocal of the interchannel frequency difference $1/\Omega$. In the leading order the amplitude of the emitted radiation is proportional to d_3/Ω^2, and the source term for this radiation is identical to the one produced by perturbation of the second-order dispersion coefficient. The only other effects up to the third order are shifts in the soliton's phase and position. Our results show that the statistical description of soliton propagation in a given channel influenced by interaction with a quasi-random sequence of solitons from other channels is similar to the description of soliton propagation in fibers with weak disorder in the second-order dispersion coefficient.

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The electrodynamics of ultrashort optical pulses in optical fibers is a rich and important field of modern nonlinear optics.[1,2] It also has a broad variety of potential industrial applications in next-generation high-speed optical fiber communications and photonic optical interconnect technologies (see, e.g., Ref. 3). Modern high-speed optical fiber communication systems make extensive use of multifrequency channel technology [wavelength division multiplexing, see, e.g., Ref. 3] for transmission of information. One of the major limitations on the performance of such systems is caused by the nonlinear interchannel interaction of optical pulses from different channels (interchannel collisions). With the increasing demand for faster transmission of information, one can expect a corresponding increase in the number of channels used and a decrease in the width of the pulses launched. As a result, the importance of interchannel collisions between optical pulses is expected to increase, and an accurate description of the effects of these collisions is needed. In this paper we study this phenomenon by using the conventional optical soliton as an example. We focus our attention on the effect of third-order dispersion on the interchannel collisions between solitons, which is expected to be dominant (in comparison with other inelastic effects) near the zero-dispersion wavelength.

In an ideal fiber, interchannel collisions between solitons can be modeled by use of the nonlinear Schrödinger equation (NLSE). In that case no radiation is emitted owing to the collisions; i.e., the collisions are elastic. The amplitude, frequency, and shape of the solitons are not changed by the collisions either. The only effects of an ideal collision are a phase shift proportional to $1/|\Omega|$ and a position shift proportional to $1/(|\Omega|\Omega)$, where Ω is the frequency difference between the solitons. In real optical fibers, however, this ideal elastic nature of soliton collisions breaks down owing to the presence of high-order corrections (perturbations), such as third-order dispersion, Raman scattering, and self-steepening, to the ideal NLSE. In this case, collisions between solitons from different frequency channels might lead to emission of radiation, change in the soliton amplitude and frequency, corruption of the soliton shape, stronger shift in the soliton position, and other undesirable effects. (See Fig. 1 for a schematic of the collision process.) In addition, the radiation emitted owing to interchannel collisions might lead to interaction between solitons from the same frequency channel (intrachannel interaction). Therefore it is important to have a realistic estimation for the intensity of the radiation emitted, as well as for the change in the soliton parameters due to interchannel collisions.

Intrachannel interaction[4–6] and interchannel collisions[7,8] between solitons of the ideal NLSE have been studied in detail and are by now well understood. Substantial progress has also been made in understanding the effects of perturbations (breaking the ideal, integrable nature of the NLSE) on intrachannel interaction.[4,9–13] In particular, several authors[9,10,14] considered the problem of soliton fission, which is the splitting of a bound two-soliton state into two separate solitons due to the presence of perturbations. In contrast, accurate analysis of the effects of perturbations on interchannel collisions is a very complicated and long-standing problem, which, to the best of our knowledge, was never fully addressed in the past. The main technical problem in this case is how to develop a perturbation theory around the multisoliton solutions of the ideal NLSE. In spite of the existence of exact expressions for the multisoliton solutions of the ideal NLSE, direct perturbative analysis around the complex multisoliton solutions has not yet been successful. It should also be mentioned that direct numerical simula-

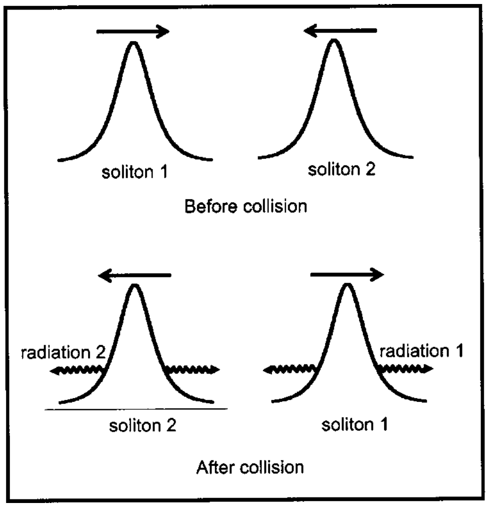

Fig. 1. Schematic description of the collision between two solitons from different frequency channels.

tion of the problem is very difficult, since an accurate measurement of the collision-induced radiation requires exact elimination (from the simulations results) of the radiation emitted owing to single-pulse propagation.

In this paper we describe in a concise but self-containing manner the solution of this long-standing problem for the case in which the perturbation is due to third-order dispersion. The effect of third-order dispersion on the interchannel collisions is expected to be dominant compared with other high-order effects near the zero-dispersion wavelength. We calculate the dynamics and the total intensity of the continuous radiation emitted as a result of the collision. We also calculate the change, induced by the collision, in the soliton parameters. To achieve these goals, we first find a single-soliton stationary (i.e., independent of the position along the fiber) solution, taking into account third-order dispersion. This stationary solution is similar to the one obtained by Kodama and Mikhailov by means of the normal form theory.[14,15] It should also be mentioned that the propagation of a single pulse in the presence of third-order dispersion was studied in detail by several authors.[15–18] It was found that even if the pulse launched into the fiber is not exactly of the stationary form it evolves into the stationary form after a transient.[17] We then use two such stationary solutions as the initial condition for the collision problem. To find the effects of the collision, we develop a perturbation theory with respect to two small parameters: the dimensionless third-order dispersion coefficient d_3 and the reciprocal of the dimensionless interchannel frequency difference $1/\Omega$. The amplitude of the emitted radiation is found to be proportional to d_3/Ω^2, and the source term for this radiation has the same form as the source term for the radiation that is emitted when a single soliton propagates under a fast change in the *second*-order dispersion coefficient. The amplitude and the phase velocity of the solitons do not acquire any change up to the third order of the perturbation theory. It is also found that soliton propagation in a given channel under many collisions with solitons from other channels is identical to propagation of solitons in a fiber with weak disorder in the second-order dispersion coefficient. (The latter problem was addressed in Ref. 13.) Notice that the major technical tool used in the analytical calculations is singular perturbation theory that is an appropriate extension of the technique developed by Kaup.[19]

These results along with the general scheme of their derivation are explained here in a brief but self-containing form. We also present here a detailed discussion of the implications of these results for some current and future (ultrafast) fiber-optics communication systems. Discussion of many technical (mathematical) details will be presented elsewhere.[20]

Propagation of short-wave packets through an optical fiber is described by the following modification of the NLSE (see Ref. 1, p. 44):

$$i\partial_z\Psi + \partial_t^2\Psi + 2|\Psi|^2\Psi = id_3\partial_t^3\Psi, \tag{1}$$

where z is the position along the fiber and t is the retarded time associated with the reference channel. Coefficients in front of the second-order dispersion term and the nonlinear Kerr term are rescaled to unity and to the factor of 2 in Eq. (1) by a proper choice of time units and Ψ units, respectively.[21] The term that appears on the right-hand side of Eq. (1) accounts for the effect of third-order dispersion (linear dependence of fiber chromatic dispersion on the wavelength of the carrier frequency), with d_3 as a constant. Higher-order terms (with higher-order temporal derivatives and other than those given by the Kerr-term types of nonlinearity) can be neglected in the majority of practical cases. Notice also that fiber losses in Eq. (1) are omitted. Equation (1) applies to the description of the interchannel interaction of optical pulses in three cases: (i) the dispersion length, length of nonlinearity, and characteristic distance of soliton interaction are much smaller than the characteristic length of fiber losses[22]; (ii) fiber losses are compensated by in-line distributed optical amplifiers; and (iii) these losses are compensated by lumped amplification achieved by insertion of fiber spans with exponentially decreasing spatial dispersion profiles[24] (dispersion-tapered fibers).

It is important to mention that Eq. (1) is generic, as it explains simultaneous propagation through many frequency channels. Unlike in the degenerate case of $d_3 = 0$, Eq. (1) is not integrable. However, for many practical examples $d_3 \ll 1$; thus a perturbative calculation about the integrable $d_3 = 0$ limit is justified.

A single-soliton solution of Eq. (1) with $d_3 = 0$ in a given frequency channel, characterized by a frequency shift Ω relative to a reference channel, is given by

$$\eta\frac{\exp[i\alpha + i\Omega(t - y) + i(\eta^2 - \Omega^2)z]}{\cosh[\eta(t - y - 2\Omega z)]}, \tag{2}$$

where α, η, and y stand for the soliton phase, amplitude, and position, respectively. Assuming that $d_3 \ll 1$, we will be looking for a stationary perturbative single-soliton solution of Eq. (1) in the form

$$\Psi_\Omega(t, z) = [\tilde{\Psi}_{\Omega 0}(h_\Omega) + \tilde{\Psi}_{\Omega 1}(h_\Omega) + \ldots]\exp(i\chi_\Omega), \quad (3)$$

where $\tilde{\Psi}_{\Omega 0} = \eta_\Omega \cosh^{-1}(h_\Omega)$, $h_\Omega = \tilde{\eta}_\Omega \tau_\Omega$, $\tau_\Omega = t - y_\Omega - 2\Omega(1 + 3d_3\Omega/2)z$, $\tilde{\eta}_\Omega = (1 + 3d_3\Omega)^{-1/2}\eta_\Omega$, and $\chi_\Omega = \alpha_\Omega + \Omega(t - y_\Omega) + [\eta_\Omega^2 - \Omega^2(1 + d_3\Omega)]z$. The first term on the right-hand side of Eq. (3) is the ideal single-soliton solution, which accounts for the shift in the second-order dispersion coefficient $\sim d_3\Omega$. One can see that this shift changes the group velocity by $3d_3\Omega/2$ and the pulse width by the factor of $(1 + 3d_3\Omega)^{1/2}$. (The shift is not necessarily small. The only limitation on $d_3\Omega$ is $1 + d_3\Omega \gg d_3$, which is the condition that the second-order dispersion coefficient in the channel Ω is to be much larger than the third-order dispersion coefficient. A similar form for $\tilde{\Psi}_{\Omega 0}$ was actually suggested in Ref. 25.) The second term in Eq. (3) is perturbative, $O(d_3)$. To calculate this term, we adopt the perturbation method introduced by Kaup in Ref. 19. In Kaup's theory, the differential operator $\hat{L}_\eta$ is used to describe a linear perturbation around the ideal soliton solution. The complete system of eigenfunctions of $\hat{L}_\eta$ includes a continuous spectrum of delocalized modes, as well as four discrete localized modes, related to small changes in the four parameters of the soliton: Ω, α, η, and y. We expand $\tilde{\Psi}_{\Omega 1}$ in terms of the eigenfunctions of $\hat{L}_\eta$ and calculate the coefficients of this expansion. It is shown in Ref. 20 that $\tilde{\Psi}_{\Omega 1}$ is stable and localized. A similar analysis was carried out by means of the normal form theory in Refs. 14 and 15. Stationarity of the solution [Eq. (3)] means that each of the solitons propagates without any change in their parameters and without shedding any radiation; thus effects of radiation emission and parameter change are due only to soliton collisions.[26]

Let us now describe collision between two solitons from different channels. For simplicity, and without any loss of generality, we choose one of the channels to be the reference one with $\Omega = 0$. We also assume that for the second channel Ω is much larger than the inverse width of the pulse, i.e., $|\Omega| \gg 1$. We are looking for a two-soliton solution of Eq. (1) in the form $\Psi_{\text{two}} = \Psi_0 + \Psi_\Omega + \Phi$, where Ψ_0 and Ψ_Ω are described by Eq. (3) with $\Omega = 0$ and Ω, respectively, and Φ is a small correction due to collision. It is straightforward to check that the exact two-soliton solution of Eq. (1) at $d_3 = 0$ acquires the form

$$\Psi_{\text{two}} = \Psi_0 + \Psi_\Omega + \Phi_0 + \Phi_\Omega + \Phi_{-\Omega} + \Phi_{2\Omega} + O(1/\Omega^3), \quad (4)$$

where Φ_0 and Φ_Ω are corrections of the leading order $1/\Omega$ in the channels 0 and Ω, respectively. The terms $\Phi_{-\Omega}$ and $\Phi_{2\Omega}$ correspond to $O(1/\Omega^2)$ corrections in channels $-\Omega$ and 2Ω, respectively; the two latter corrections are exponentially small outside the collision region. By analogy with the ideal $d_3 = 0$ case, one substitutes a solution of the form [Eq. (4)] into Eq. (1) and calculates Φ_0. Since Φ_0 oscillates together with Ψ_0 and $|\Omega| \gg 1$, one neglects the exponentially small contributions from the terms rapidly oscillating with t and z. Then the equation describing Φ_0 is

$$\begin{aligned}\partial_z\tilde{\Phi}_0 - i[(\partial_t^2 - \eta_0^2)\tilde{\Phi}_0 + 4|\Psi_0|^2\tilde{\Phi}_0 + 2\tilde{\Psi}_0^2\tilde{\Phi}_0^*] \\ = 4i[|\Psi_\Omega|^2\tilde{\Psi}_0 + |\Psi_\Omega|^2\tilde{\Phi}_0 + \tilde{\Psi}_0(\Psi_\Omega\Phi_\Omega^* + \Psi_\Omega^*\Phi_\Omega) \\ + \tilde{\Psi}_0^*\Psi_\Omega\Phi_{-\Omega} + \tfrac{1}{2}\Psi_\Omega^2\Phi_{2\Omega}^* + \tilde{\Psi}_0|\Phi_0|^2 + \tfrac{1}{2}\tilde{\Psi}_0^*\tilde{\Phi}_0^2 \\ + \tilde{\Psi}_0|\Phi_\Omega|^2 + \tilde{\Phi}_0(\Psi_\Omega\Phi_\Omega^* + \Psi_\Omega^*\Phi_\Omega)] + d_3\partial_t^3\tilde{\Phi}_0,\end{aligned} \quad (5)$$

where $\tilde{\Phi}_0 \equiv \Phi_0\exp(-i\chi_0)$ and $\tilde{\Psi}_0 \equiv \Psi_0\exp(-i\chi_0)$. Vicinity (in z) of the collision event is given by $[z_0 - \tilde{z}/|\Omega|, z_0 + \tilde{z}/|\Omega|]$, where $|\Omega| \gg \tilde{z} \gg 1$, and is naturally separated from the regions before and after the collision. In the collision region, $\tilde{\Phi}_0$ acquires a fast (with respect to z) change. Since for this region $\Delta z \sim 1/\Omega$, the $\partial_z\Phi_0$ and $|\Psi_\Omega|^2\tilde{\Psi}_0$ terms give leading contributions to Eq. (5), whereas the $\partial_t^2\tilde{\Phi}_0$ term can be neglected together with all the other terms. In the successive orders of the perturbation theory, one should carefully consider contributions coming from terms such as $\partial_t^2\tilde{\Phi}_0$ and $d_3\partial_t^3\tilde{\Phi}_0$. In the two other regions, $z < z_0 - \tilde{z}/|\Omega|$ and $z > z_0 + \tilde{z}/|\Omega|$, the interaction between the solitons is exponentially small, so that all the interaction terms there can be neglected. Formally, separation into three well-defined regions means that one can replace all the terms in Eq. (5), except for $\partial_z\tilde{\Phi}_0$, with $C\delta[\Omega(z - z_0)]$, where $\delta(z)$ is the Dirac delta function and the constant C is simply the integral of all these terms over z. This separation results in the three well-formulated Cauchy problems for $\tilde{\Phi}_0$ in the three regions.

Calculating $\tilde{\Phi}_0$, one distinguishes between two types of collision-induced corrections to the stationary solutions [Eq. (3)]. The first one corresponds to changes in the pulse parameters on top of the stationary solution, and the second one corresponds to deformations of the form of Eq. (3), which cannot be reduced to changes in the pulse parameters, and thus leads to emission of radiation. Even though the rigorous calculation of the effects of the collision in successive orders of the perturbation theory is quite complicated,[20] the main result can be derived in a straightforward manner by use of just a few equations. We start by noticing that the equation for the $O(1/\Omega)$ collision-induced correction $\tilde{\Phi}_{01}^{(0)}$ is

$$\partial_z\tilde{\Phi}_{01}^{(0)} = 4i|\Psi_{\Omega 0}|^2\tilde{\Psi}_{00} = \frac{4i\,\eta_0\eta_\Omega^2}{\cosh(h_0)\cosh^2(h_\Omega)}. \quad (6)$$

In Eq. (6) the first subscript in $\tilde{\Phi}_{01}^{(0)}$ stands for the 0 channel, the second subscript stands for the total combined order in both d_3 and $1/\Omega$, and the superscript stands for the corresponding order in d_3. Integrating Eq. (6) from $-\infty$ to some general z, one obtains

$$\tilde{\Phi}_{01}^{(0)}(t, z) = -\frac{2i\,\eta_0\eta_\Omega(1 + 3d_3\Omega)^{1/2}}{(1 + 3d_3\Omega/2)\Omega}\frac{\tanh(h_\Omega)}{\cosh(h_0)}. \quad (7)$$

The source term for the emitted radiation $\tilde{\Phi}_{03}^{(1)R}$ is of order d_3/Ω^2. The equation for this source term is

$$\partial_z \tilde{\Phi}_{03}^{(1)R} = d_3 \partial_t^3 \tilde{\Phi}_{01}^{(0)}. \tag{8}$$

Substituting Eq. (7) into Eq. (8) and integrating over the collision region, one obtains

$$\tilde{\Phi}_{03}^{(1)R}(t, z_0 + \tilde{z}/|\beta|) = -iB\partial_{h_0}^2 \tilde{\Psi}_{00}(h_0), \tag{9}$$

where the coefficient B is defined by

$$B = \frac{6\eta_0^3 \eta_\Omega (1 + 3d_3\Omega)^{1/2} d_3}{(1 + 3d_3\Omega/2)^2 \Omega|\Omega|}. \tag{10}$$

From Eq. (9) it follows that the source term for the emitted radiation has the same form as the source term for the radiation that is emitted when a single soliton propagates under a fast change (with respect to z) in the second-order dispersion coefficient. Once the source term $\tilde{\Phi}_{03}^{(1)R}$ is calculated, one can also calculate the total radiation energy $\mathcal{E}_0$ emitted by the reference channel soliton:

$$\mathcal{E}_0 \simeq \frac{192\eta_0^5 \eta_\Omega^2 (1 + 3d_3\Omega) d_3^2}{5(1 + 3d_3\Omega/2)^4 \Omega^4}. \tag{11}$$

One also finds that the only changes in the pulse parameters up to the third order of the theory are the $O(1/\Omega)$ phase shift, $\Delta\alpha_0 \simeq 4\eta_\Omega(1 + 3d_3\Omega)^{1/2}[(1 + 3d_3\Omega/2)|\Omega|]^{-1}$, and the $O(1/\Omega^2)$ position shift, $\Delta y_0 \simeq -4\eta_\Omega(1 + 3d_3\Omega)^{1/2}[(1 + 3d_3\Omega/2)^2\Omega|\Omega|]^{-1}$, which are already present in the ideal ($d_3 = 0$) collision case. It should be mentioned that a similar expression for Δy_0 (with $d_3 = 0$) was obtained by Mollenauer *et al.*[7,24] for the ideal collision case. It is also interesting to note that the $O(d_3)$ stationary solution $\tilde{\Psi}_{\Omega 1}$ behaves like an ideal soliton in the collision: It acquires the $\Delta\alpha_0$ phase shift and the $\eta_0\Delta y_0$ position shift and contributes to radiation only in the order d_3^2/Ω^2 of the theory. The soliton amplitude and phase velocity do not acquire any change up to third order of the theory. The result for the soliton amplitude is consistent with the conservation law for the total energy, which requires $\eta = 1 + O(d_3^2/\Omega^4)$ for both solitons. (See also Ref. 13 for a discussion of a similar situation induced by the fluctuation of the second-order dispersion coefficient.)

Let us use our results to make specific predictions for an optical fiber setup with distributed amplification compensating losses or with lumped amplification and dispersion-tapered fibers. Taking $\eta_0 = 1$ and requiring that the widths of the two solitons be equal (bit rates should be the same in all channels), one obtains $\eta_\Omega = (1 + 3d_3\Omega)^{1/2}$. We take the channel spacing Ω as $\Omega = \Delta\nu/\nu_0 = 5$, which is the typical value used in current wavelength-division-multiplexing systems, operating at a bit rate of 10 Gbit/s.[24,27,28] Indeed, in these systems the typical pulse width is $\tau_0 = 20$ ps, corresponding to $\nu_0 = 15$ GHz, and the channel spacing is 0.6 nm, corresponding to $\Delta\nu = 75$ GHz. The error in the calculation of the energy loss due to collision-induced emission of radiation can be estimated as $\sim 1/\Omega$, which for $\Omega = 5$ is approximately 20%. We consider multichannel transmission near the zero-dispersion wavelength, where the effect of third-order dispersion on soliton collisions is expected to be the most significant. We point out that the single-channel soliton transmission at the zero-dispersion wavelength was considered by many authors[29–31] because of its potential advantages, e.g., the low power required for the transmission. By use of the definitions of d and d_3 in Ref. 21, it is easy to see that for $\Omega = 5$ our perturbation theory is applicable already to the channel next to the zero-dispersion channel. Indeed, in this case $|\beta_2| = (10\beta_3)/(\pi\tau_0)$, and therefore $d_3 = \beta_3/(3|\beta_2|\tau_0) \sim 0.1$ for this channel. Let us choose this (closest to the zero-dispersion point) channel to be the reference channel. We then find that the fraction of energy emitted in the form of radiation by a soliton from the reference channel, as a result of a collision with a soliton from the next channel (the channel separated from the zero-dispersion channel by $2\Omega = 10$) is 2×10^{-4}. The collision also leads to a corresponding decrease in the amplitude and increase in the width of the soliton by a factor of 2×10^{-4}. Even though this is a relatively small effect, the accumulative effect of many collisions taking place while the soliton passes a large distance along the fiber might be very destructive. To estimate this effect, we first calculate the average distance Δx_{ic} passed by the soliton in the reference channel between two successive collisions. Using the relations in Ref. 21, one obtains

$$\Delta x_{ic} = \frac{\pi T \tau_0^3}{2(1 + 3d_3\Omega/2)\beta_3|\Omega|^2 s}, \tag{12}$$

where T is the time-slot width and s is the fraction of occupied time slots. Typical values are $T = 5$, $s = 0.5$, and $\beta_3 = 0.1\ \mathrm{ps}^3/\mathrm{km}$. Consider, for example, a system operating at a bit rate of 160 Gbit/s and with pulse widths of 1 ps. Long-distance single-channel transmission experiments at 80 Gbit/s[32] and at 160 Gbit/s[33] have shown that soliton transmission at such high bit rates are possible. Additional experiments[34] have shown that transoceanic multichannel transmission at 80 Gbit/s per channel is also possible. Therefore one can consider the possibility of implementing multichannel transmission with bit rates as high as 160 Gbit/s. For the system considered in our example the average distance passed by the soliton until it experiences 1000 collisions and loses approximately 20% of its energy is 4500 km. If we take into account collisions with solitons in the zero-dispersion channel (estimating their contribution to be similar) and an extreme case, in which $s \sim 1$, this distance decreases to approximately 1000 km. (The energy loss due to single-pulse propagation, which is proportional to d_3^2, is only 1% in this case.) For systems operating at lower bit rates (and higher pulse widths), the average distance between collisions increases like τ_0^3 does, and the accumulative effect of the collisions decreases correspondingly.

Another accumulative effect, which can be even more severe, is the interaction between solitons propagating in the same frequency channel induced by multiple collisions with solitons from all other channels. To study this effect, one should consider the solitons in all other channels as a pseudorandom sequence of pulses. Then propagation of solitons in a given channel is described by a perturbed NLSE, in which the perturbative term has the form of the radiation source term $\tilde{\Phi}_{03}^{(1)R}$ [given by Eq. (9)], multiplied by a function $\xi(z)$, corresponding to the random short-correlated nature of the multiple interchannel interactions. It is very important to notice that this perturbative term has the same form as the perturbative term appearing in the equation that describes the propagation of solitons in fibers with weak noise in the dispersion coefficient. This observation means that the results obtained in Ref. 13 for weakly disordered optical fibers can be directly applied to describe soliton propagation under multiple interactions with solitons from other channels. Thus one should expect the emergence of long-range radiation-mediated intrachannel interaction in multichannel systems as well.

We conclude by pointing out that this study suggests a general recipe for investigating a variety of interchannel interaction phenomena. For this purpose, one should first obtain a stationary single-soliton solution of the perturbed nonlinear Schrödinger equation. This solution then serves as the initial condition in the collision problem. Using the double-perturbation theory presented here, one can describe all effects caused by the collision. An interesting example is provided by fast soliton collisions in the presence of Raman scattering, the phenomenon that should be dominant for very short pulses.

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22. Effects of fiber losses can be neglected (see, for instance, Ref. 23) if values of $\Delta\nu$, β_2, β_3, and τ_0 satisfy the following two conditions: $\tau_0 \ll \sqrt{\beta_2/\gamma}$ and $\Delta\nu \gg (\gamma\tau_0^2)/\beta_3$.
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