Sample problems for review for the Final Exam

1. Write the number in the form $a+\mathrm{i}b$.

(a) $\dfrac{1}{3+4\mathrm{i}}$

(b) $\mathrm{Log}(\mathrm{i})$

(c) $(1-\mathrm{i})^{16}$

(d) $\exp(2-3\mathrm{i})$

(e) $\arctan\left(\dfrac{1+\mathrm{i}}{\sqrt{2}}\right)$, give all values $\left(\text{hint: } \arctan(z) = \dfrac{\mathrm{i}}{2}\log\dfrac{1-\mathrm{i}z}{1+\mathrm{i}z}\right)$

2. Find all the values of $(-64)^{1/6}$. Sketch the "values of $(-64)^{1/6}$"="solutions of $z^6+64=0$" on the complex plane.

3. Check where the Cauchy–Riemann equations for the function $f(z)$ hold ($z = x + \mathrm{i}y$).

(a) $f(z) = \dfrac{x}{x^2+y^2} - \mathrm{i}\dfrac{y}{x^2+y^2}$

(b) $f(z) = x^3 - 3xy^2 + \mathrm{i}y^3 - 3\mathrm{i}x^2y$

4. Find the radius of convergence of the power series

(a) $\displaystyle\sum_{n=0}^{\infty} n^{2008} z^n$

(b) $\displaystyle\sum_{n=0}^{\infty} (12-5\mathrm{i})^n z^{2n}$

(c) Taylor series for the function $(2+z)^{1/3}$ about $z=0$ $\left(\text{which goes as } 2^{1/3}+\frac{z}{2^{2/3}3}-\frac{z^2}{2^{2/3}18}+\ldots\right)$

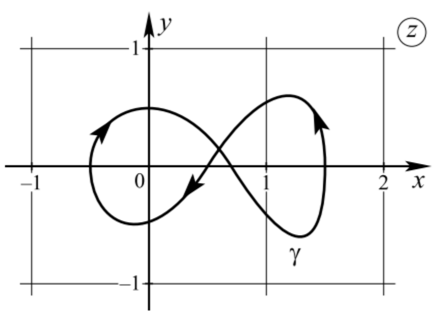
5. Evaluate the integrals

(a) $\int\limits_{-\infty}^{\infty}\frac{dx}{(1+x^2)^3}$

(b) $\oint\limits_{|z|=2}\frac{\cos(\pi z)}{z(z-1)}dz$

(c) $\oint\limits_{|z|=11}\frac{\sin z}{z^2}dz$

(d) $\int\limits_{\gamma}\frac{dz}{z(z^2-1)}$



6. Evaluate integrals of trigonometric functions over $[0,\pi]$ and integrals involving fractional powers

$$(a)\quad \int_0^{2\pi}\frac{d\theta}{2+\cos^2\theta}$$

$$(b)\quad \int_0^{2\pi}\frac{d\theta}{(2-\sin\theta)^2}$$

(c) $\int_0^\infty \frac{\sqrt{x}}{x^2+2x+5}dx$

7. Find the Laurent series for the given functions about the indicated point.

(c) $\exp\left(\frac{1}{z-1}\right)$; $z_0 = 1$

(b) $\frac{1}{z^2+1}$; $z_0 = 0$, write an expansion that works for $|z - z_0| > 1$

(c) $\sqrt{1+\sin z}$ $z_0 = 0$ (three first non-zero terms of the (in this case) Taylor series)

8. Find the linear fractional transformation that maps the triple $(-2, 0, i)$ to the triple $(3, 1, \frac{3+i}{2})$.

9. Find $f(z)$ that maps the half plane $U = \{z: Im\ z > 0\}$ onto the disk $\Delta = \{w: |w| < 1\}$

10. Find $f(z)$ that maps the strip $\pi \leq y \leq \pi$ onto the punctured plane, those w with $w \neq 0$

11. See examples 3, 4, 5 page 226-227