Electro-mechanical Properties of Bones

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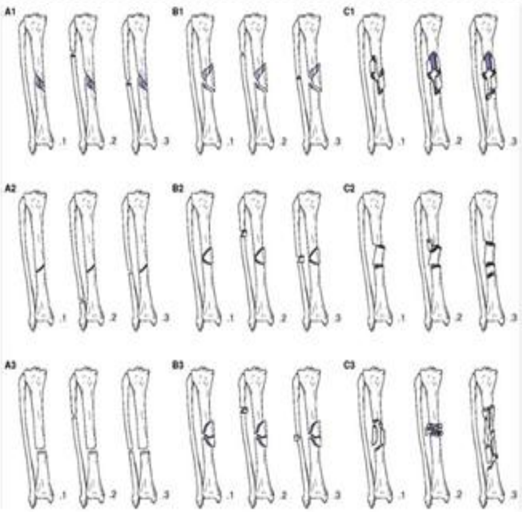
Introduction

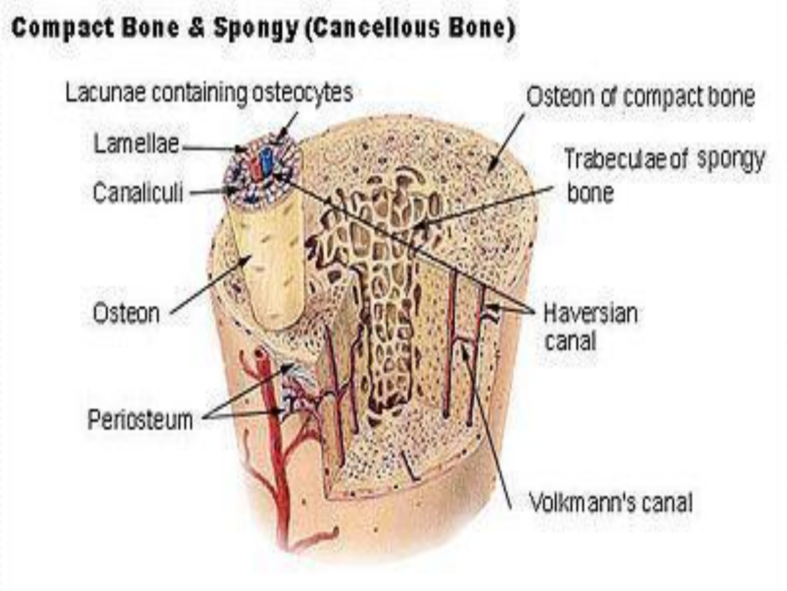
- The role of bones in the human body
  - Support/Protection
  - Production of blood cells
- The study of bone structures
  - Conductive properties of bone
- The properties of piezoelectric materials
- Applicable potential of piezoelectric property of bone
  - Aids in healing process
- Motivation of model



Properties and Structure of Bones

- Bones contain canals
  - Haversian
  - Volkmann

- Properties of the canals
  - Density/Fill Factor
  - Conductivity



Modeling the Structure

- Two ways to simplify the structure
  - Figure 1: Conductive ellipsoids running horizontally and vertically, with no interaction between them, in an isotropic medium
  - Figure 2: Conductive ellipsoids in a slightly conductive medium

- Pros/Cons of each model

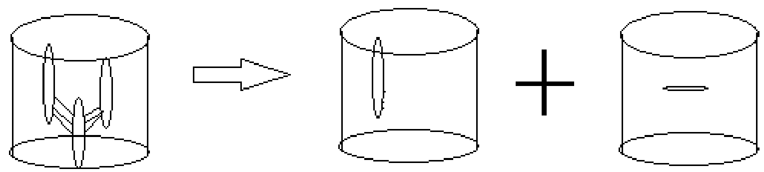


Figure 1

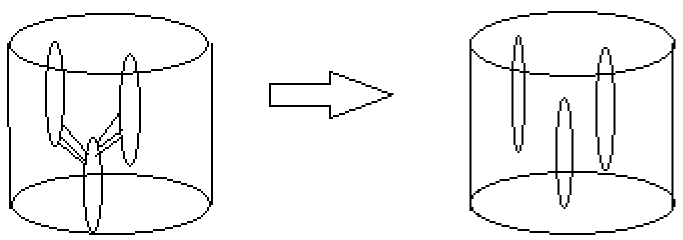
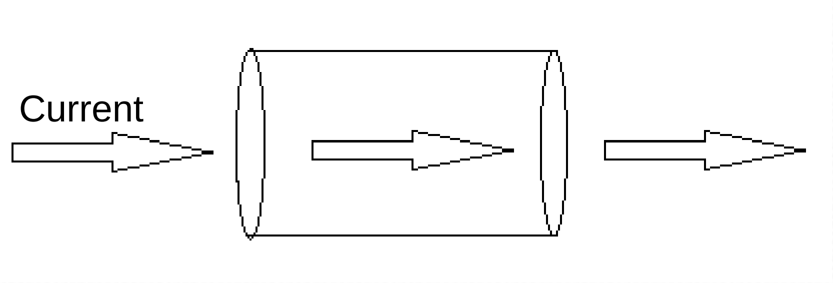


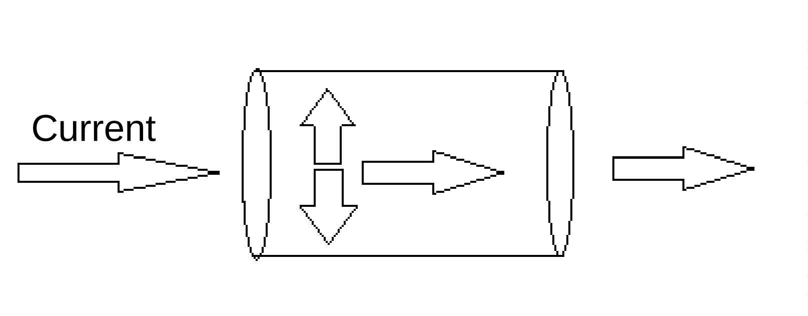
Figure 2

Current Flow Through Bones

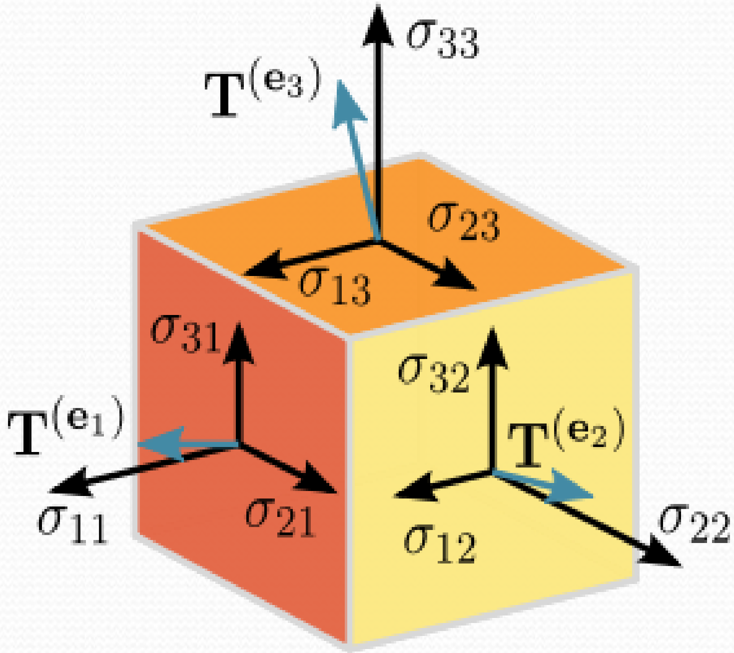
- First consider the simplest case, current through an isotropic metal rod.



- However since bones are anisotropic the flow of current is not so straightforward.

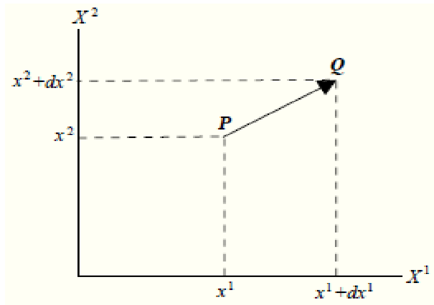


Tensors



$$\sigma = \left[\mathbf{T}^{(\mathbf{e}_1)} \mathbf{T}^{(\mathbf{e}_2)} \mathbf{T}^{(\mathbf{e}_3)}\right] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Types of Tensors



- Contravariant Tensor
  - Geometrically describes transformation of a displacement vector in one coordinate space to another.

$$d\bar{x}^1 = \frac{\partial \bar{x}^1}{\partial x^1} dx^1 + \frac{\partial \bar{x}^1}{\partial x^2} dx^2$$
$$d\bar{x}^2 = \frac{\partial \bar{x}^2}{\partial x^1} dx^1 + \frac{\partial \bar{x}^2}{\partial x^2} dx^2.$$

$$\longrightarrow \quad d\bar{x}^q = \frac{\partial \bar{x}^q}{\partial x^p} dx^p. \quad \longrightarrow \quad \bar{a}^i = \frac{\partial \bar{x}^i}{\partial x^j} a^j.$$

- Covariant Tensor
  - Geometrically describes transformation of a gradient vector in one coordinate space to another.

$$\frac{\partial \phi}{\partial \bar{x}^p} = \frac{\partial \phi}{\partial x^r} \frac{\partial x^r}{\partial \bar{x}^p}. \quad \longrightarrow \quad \bar{\nabla} \phi_p = \nabla \phi_r \frac{\partial x^r}{\partial \bar{x}^p}. \quad \longrightarrow \quad \bar{T}_p = \frac{\partial x^r}{\partial \bar{x}^p} T_r.$$

Properties of Tensors

- Inner Product

$$\mathbf{u} \cdot \mathbf{v} = u_i \, v^i$$

- Einstein Summation Convention

$$y = c_i x^i \,. \quad \longrightarrow \quad y = \sum_{i=1}^{3} c_i x^i = c_1 x^1 + c_2 x^2 + c_3 x^3$$

- Cross Product

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} \quad \longrightarrow \quad (\mathbf{u} \times \mathbf{v})_i \quad = \epsilon_{i\,j\,k} \, u^j \, v^k$$

Model of the Conductivity of Bones

- Electricity & current density
  - Dielectric material; conductive inhomogeneities

$$\Delta \boldsymbol{E} = \frac{V^*}{V_0} \boldsymbol{R} \cdot \boldsymbol{J}$$

$$\Delta \boldsymbol{J} = \frac{V^*}{V_0} \boldsymbol{K} \cdot \boldsymbol{E}$$

- Eshelby tensors
  - Resistivity & conductivity tensors

$$\boldsymbol{K} = k_0 \left(\boldsymbol{s}^C - \frac{k_0}{k_1 - k_0} \boldsymbol{I} \right)^{-1}$$

Tensors In Model

- Conductivity tensor
  - Shape factor

$$\boldsymbol{s}^C = f_0(\boldsymbol{I} - \boldsymbol{nn}) + (1 - 2f_0)\boldsymbol{nn}$$

$$f_0 = \frac{\gamma^2(1 - g)}{2(\gamma^2 - 1)}$$

- Shape Factor
  - Two cases

$$g = \begin{cases} \frac{1}{\gamma\sqrt{1-\gamma^2}} \arctan \frac{\sqrt{1-\gamma^2}}{\gamma}, & \text{oblate shape } (\gamma < 1) \\ \frac{1}{\gamma\sqrt{\gamma^2-1}} \ln\left(\gamma + \sqrt{\gamma^2 - 1}\right), & \text{prolate shape } (\gamma > 1) \end{cases}$$

Dimensionless Factors

- K tensor for conductivity
  - Revised using dimensionless factors

$$\boldsymbol{K} = -k_0(A_1\boldsymbol{I} + A_2\boldsymbol{nn})$$

- Dimensionless factors

$$A_1 = \frac{k_0 - k_1}{k_0 + (k_1 - k_0)f_0}, \quad A_2 = \frac{(k_0 - k_1)^2(1 - 3f_0)}{[k_1 - 2(k_1 - k_0)f_0][k_0 + (k_1 - k_0)f_0]}$$

- Assuming k_1 >>> k_0

$$A_1 = \frac{-1}{f_0}, \quad A_2 = \frac{1 - 3f_0}{f_0(1 - 2f_0)}$$

Results

	A_1	A_2
Single prolate spheroid $\gamma = 120$	-2.001	-3.211×10^3
Single prolate spheroid $\gamma = 80$	-2.001	-1.568×10^3
Single oblate spheroid $\gamma = 0.2$	-8.016	6.683
	$\sum R_{11} = \sum R_{22}$	$\sum R_{33}$
Haversian canals	-2.001	-3.213×10^3
Osteocyte lacunae	-4.674	-8.016
Canaliculi and Volkman's canals	-7.86×10^2	-2.001

Future Work

There are two considerations for future work

- Modeling the flow of current through a bone after homogenization of the matrix

- Changing the model in the paper to observe if neighboring Haversian canals have an effect on one another.

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