ENERGY FLOW IN ELECTRICAL GRIDS

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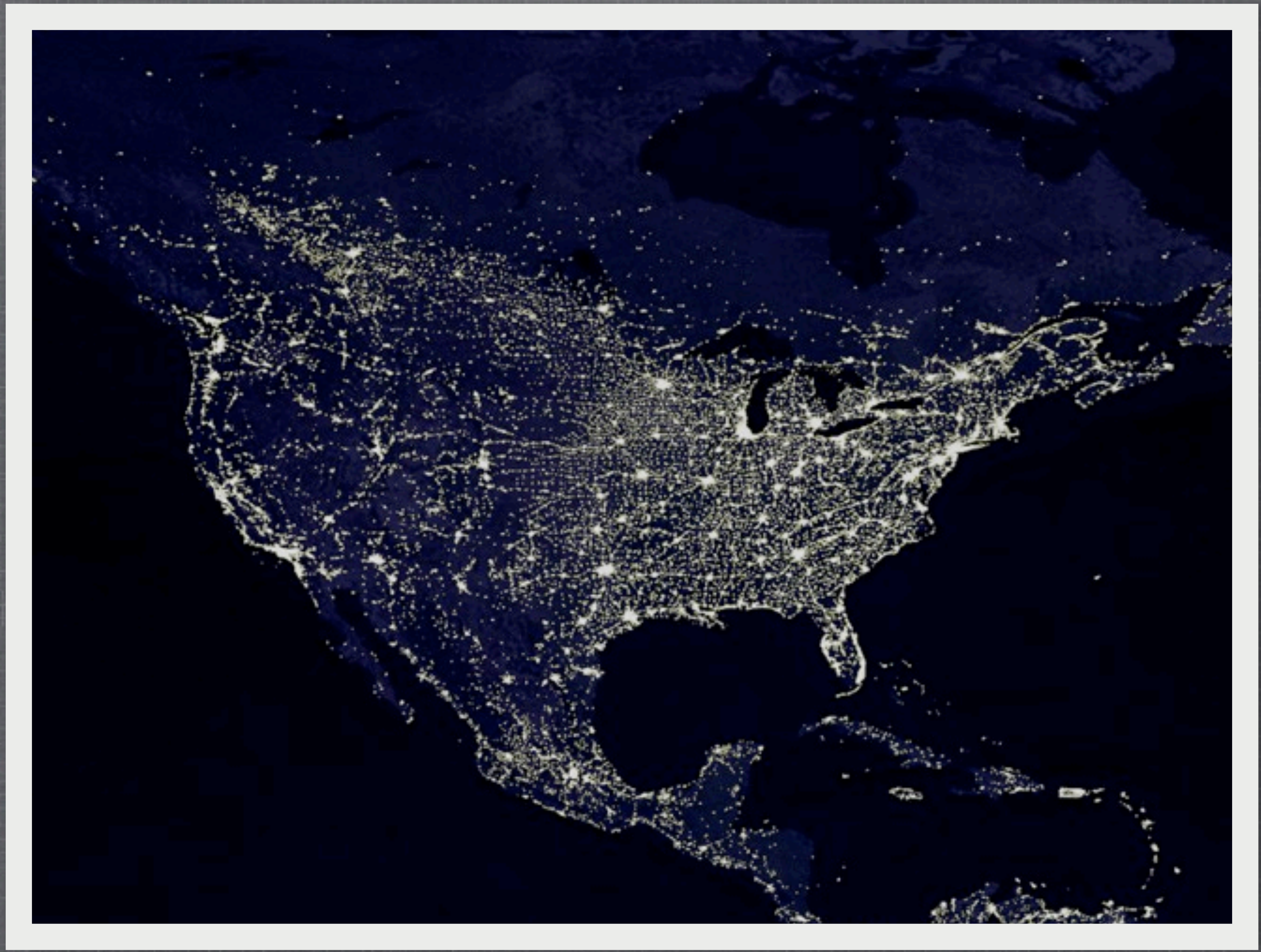

OVERVIEW

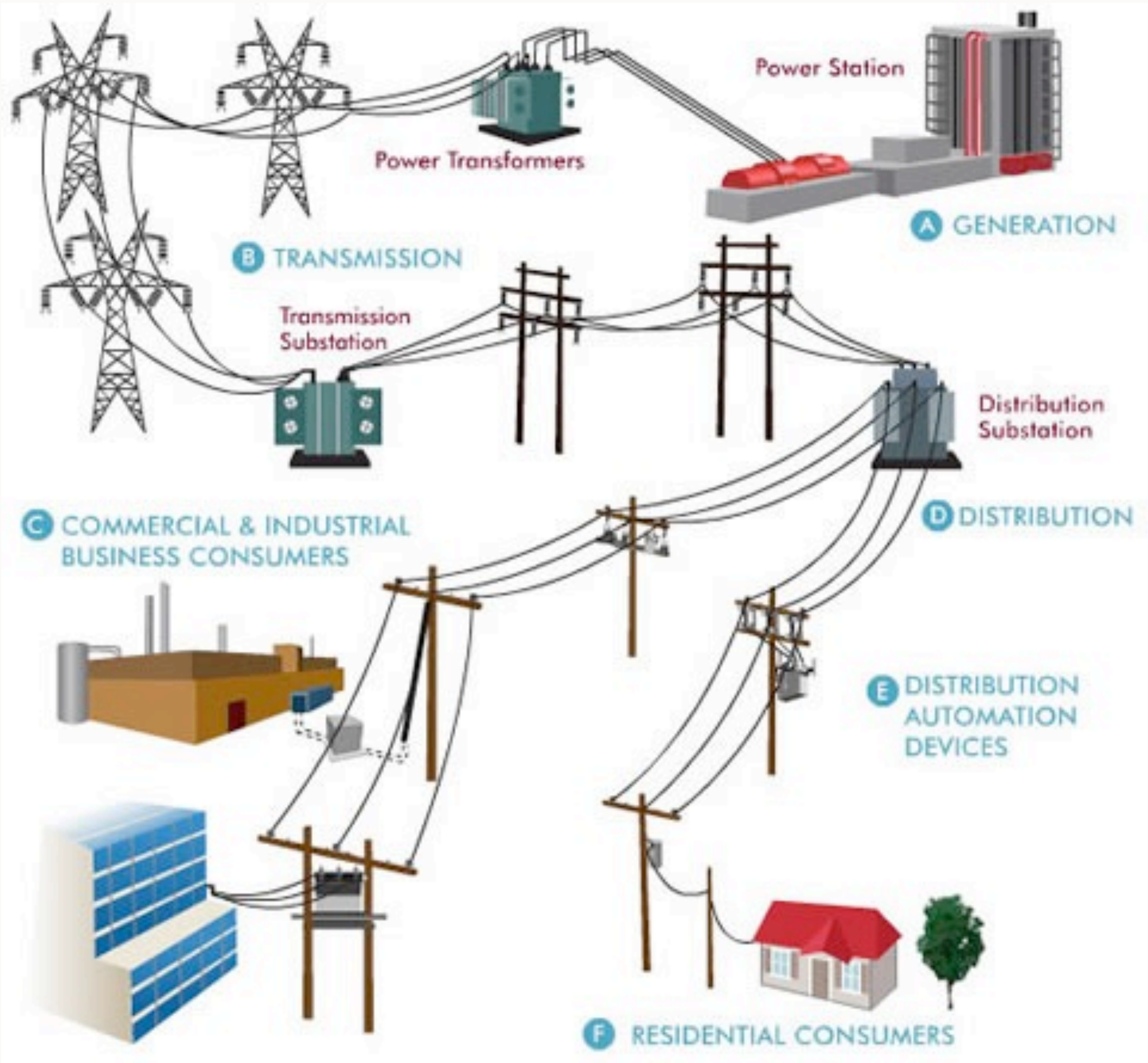

Image Source: http://www.altenergymag.com/articles/09.04.01/smartgrid/grid.jpg

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MOTIVATION

- Along a feed line voltage fluctuates
  - Desire: Reduce voltage drop at end of line
- Determine maximum length supporting consumption
- Goal: Create low-parametric ODE Model of a feeder

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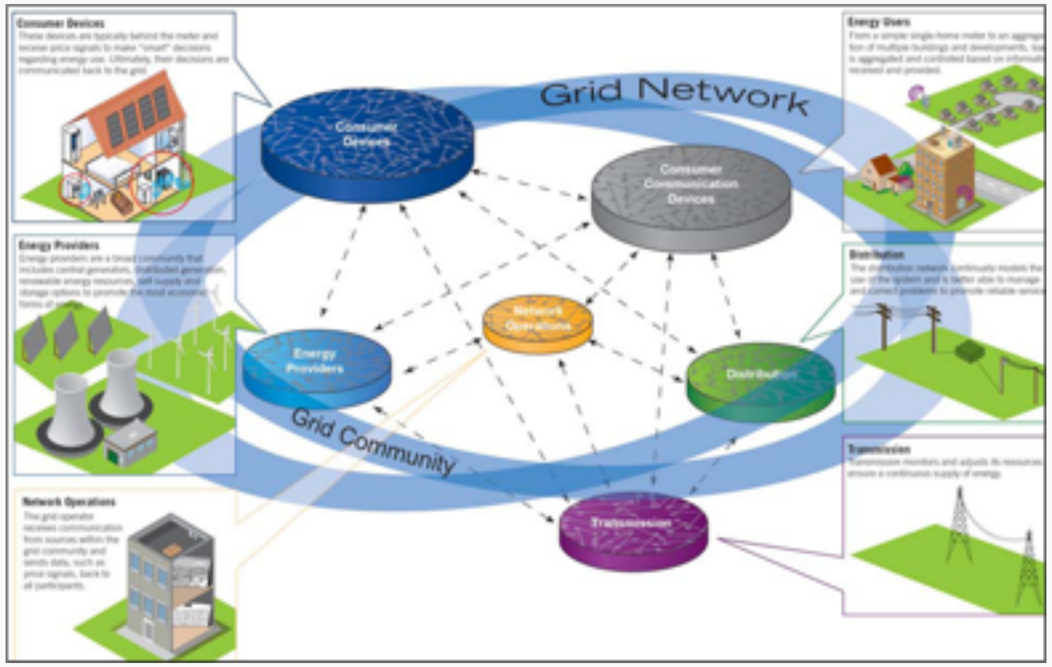
APPLICATIONS



Smart Grids

PV systems

Transporting Energy

Superbowl Power Outage

Image Sources:

http://deepresource.files.wordpress.com/2012/04/smart-grid-concept.png
http://assets.inhabitat.com/wp-content/uploads/sunpower_main.jpg
http://energy.gov/sites/prod/files/styles/topic_hero/public/Wind.jpg?itok=uYcFe4cS
http://www.mediaite.com/wp-content/uploads/2013/02/cbs-power.jpg

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BACKGROUND

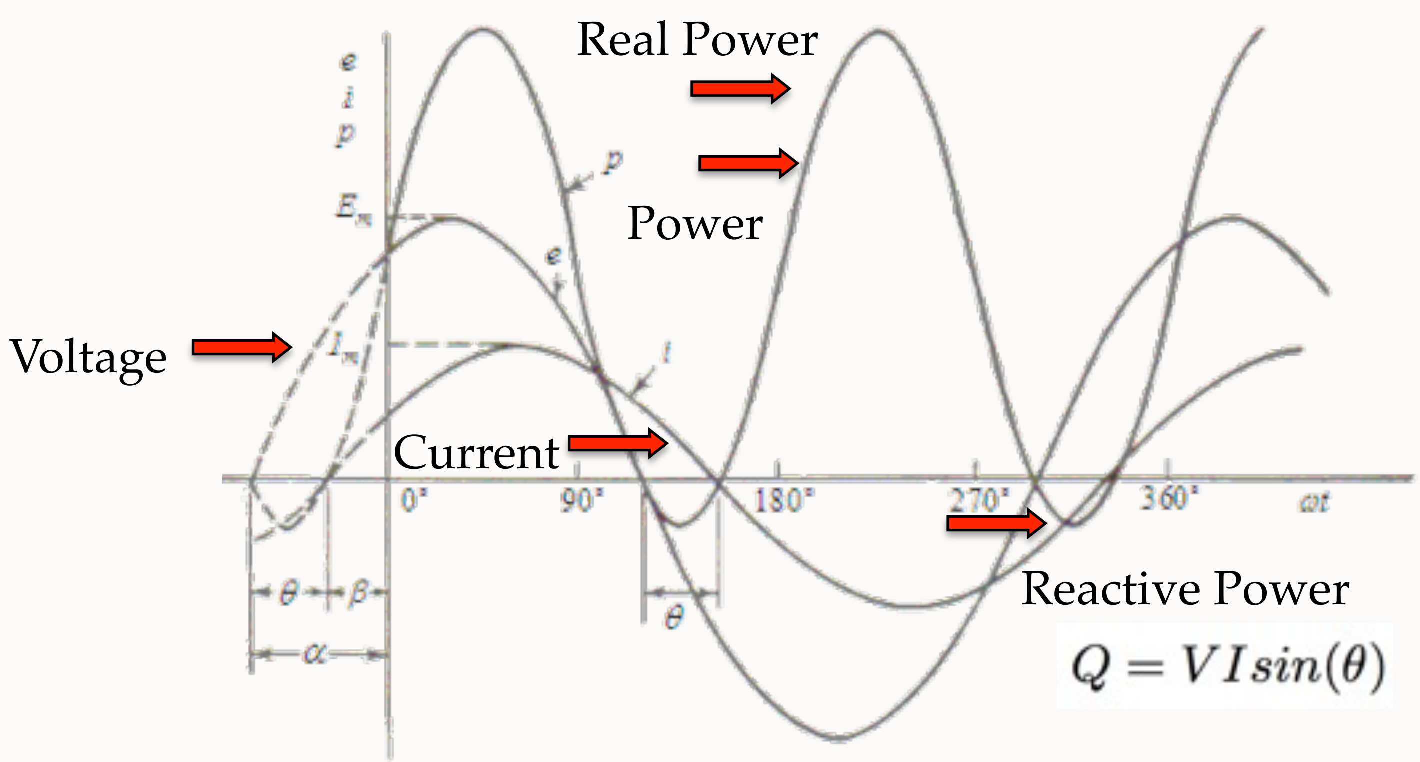
- Alternating Current

$$v(t) = \sqrt{2}V sin(\omega t + \alpha) \quad i(t) = \sqrt{2}I sin(\omega t + \beta)$$

$$p(t) = VIcos(\alpha - \beta) - VIcos(2\omega t + \alpha + \beta)$$

v = Voltage
i = Current
p = Power

$$P = VIcos(\theta)$$



$$Q = VIsin(\theta)$$

Image Source: http://prismglow.com/electronics/eees/ch02/ch2_2/inst-e,i,p-inaccircuit.gif

BACKGROUND

- Other Basic Equations

S = Apparent Power
z = impedance
x = inductance
r = resistance

$$S = VI = P + jQ$$

$$z = r + jx$$

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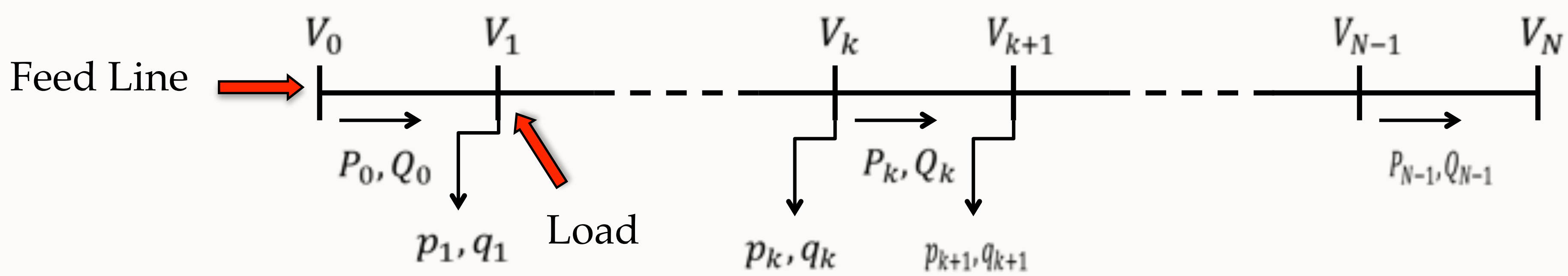
SET-UP



$$S_1 = S_0 - S_l - S_L$$

$$V_1 = V_0 - z_1 I_0$$

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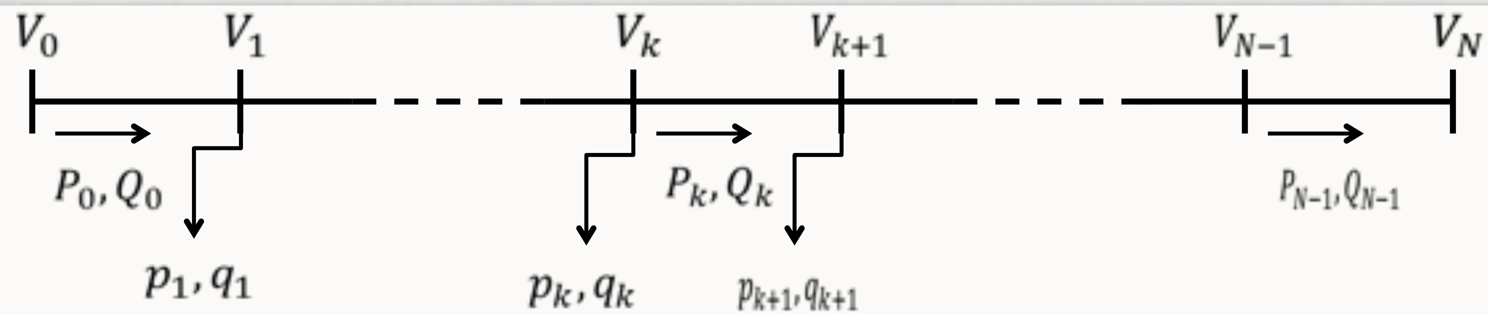
DEVELOPING THE MODEL

PROBLEM FORMULATION



Discrete form

$$\begin{aligned} P_{k+1} - P_k &= p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2} \\ Q_{k+1} - Q_k &= q_k - x_k \frac{P_k^2 + Q_k^2}{v_k^2} \\ v_{k+1}^2 - v_k^2 &= -2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2) \frac{P_k^2 + Q_k^2}{v_k^2} \end{aligned}$$

where

$k = 0, ..., N$ enumerates buses of the feeder

P_k,Q_k real and reactive power flowing from bus k to bus $k+1$

p_k, q_k overall consumption of real and reactive power at bus k

r_k, x_k line resistance and reactance connecting bus k to bus $k+1$

with Boundary Conditions

$$v_0 = 1, P_N = Q_N = 0$$

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Continuous and Homogeneous form

Large number of consumers $N \gg 1$
continuous form with limit $N \longrightarrow \infty$
$\frac{r_k}{x_k}$ is set constant. $r_k = r\frac{l_k}{L}$ and $x_k = x\frac{l_k}{L}$
L total length of the feeder line
l_k length of line from bus k to bus $k+1$
$L_k = \sum_{i=0}^{k-1} l_k$
$z = \frac{L_k}{L}$
decomposing $F_k = F(z) + \tilde{F}(L_k)/N$
p_k and q_k are small varying fast
$p(z) = p_k\frac{L}{l_k}$ and $q(z) = q_k\frac{L}{l_k}$ are in $O(1)$ and varying smoothly
Relating Finite difference to derivatives $F_{k+1} - F_k \approx F'(z)l_k/L$

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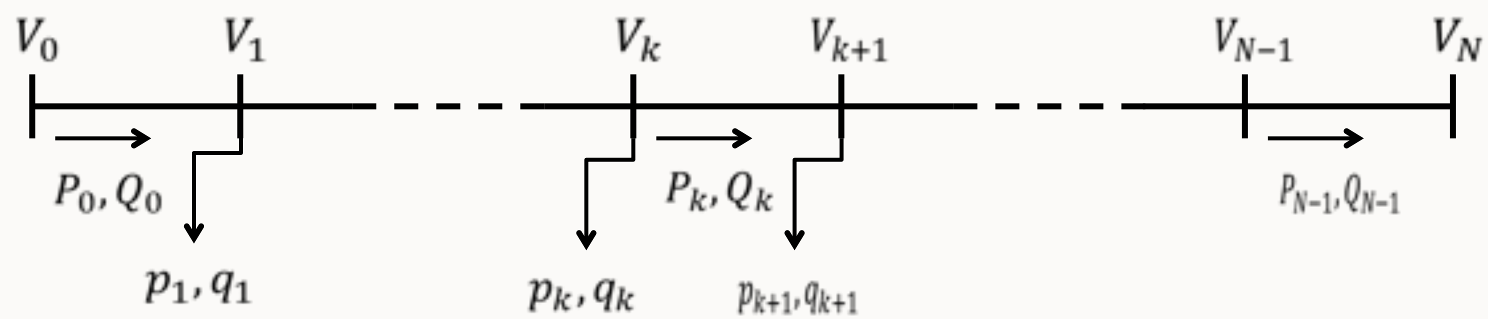

Boundary Value Problem

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{P^2+Q^2}{v^2} \\ q - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix}$$

with Boundary Conditions

$$v_0 = 1, P(L) = Q(L) = 0$$

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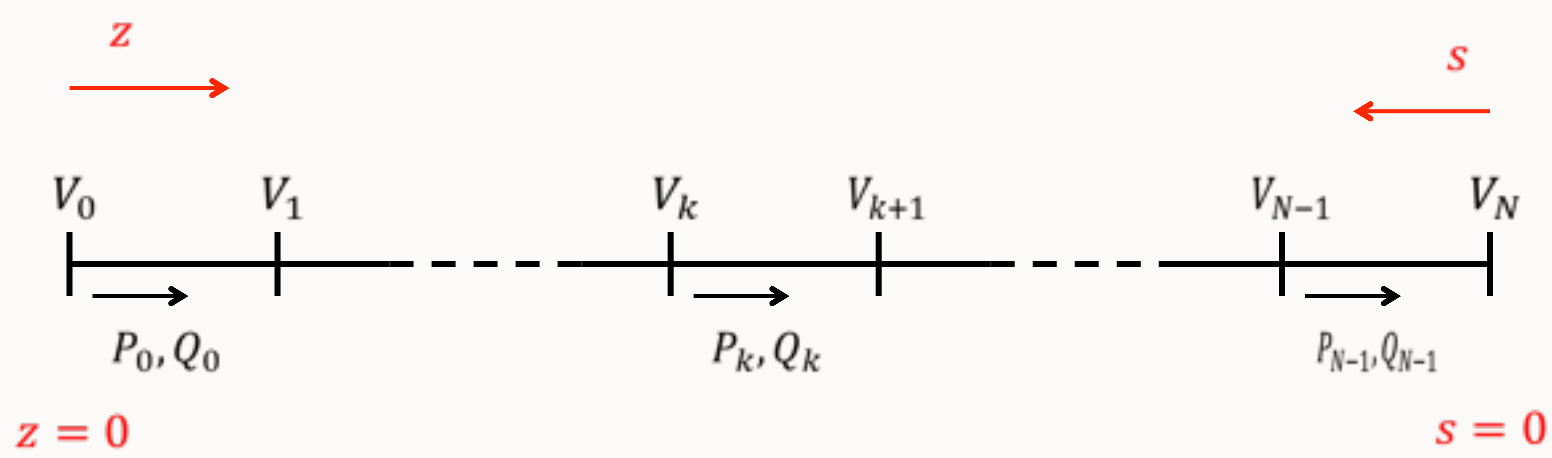


Re-scaling

Assuming p =constant and with new variable $s = \frac{\sqrt{|p|r}}{v(L)}(L - z)$

$$\varrho(s) = \sqrt{\frac{r}{|p|}} \frac{P(z)}{v(L)}$$

$$\tau(s) = \sqrt{\frac{r}{|p|}} \frac{Q(z)}{v(L)}$$

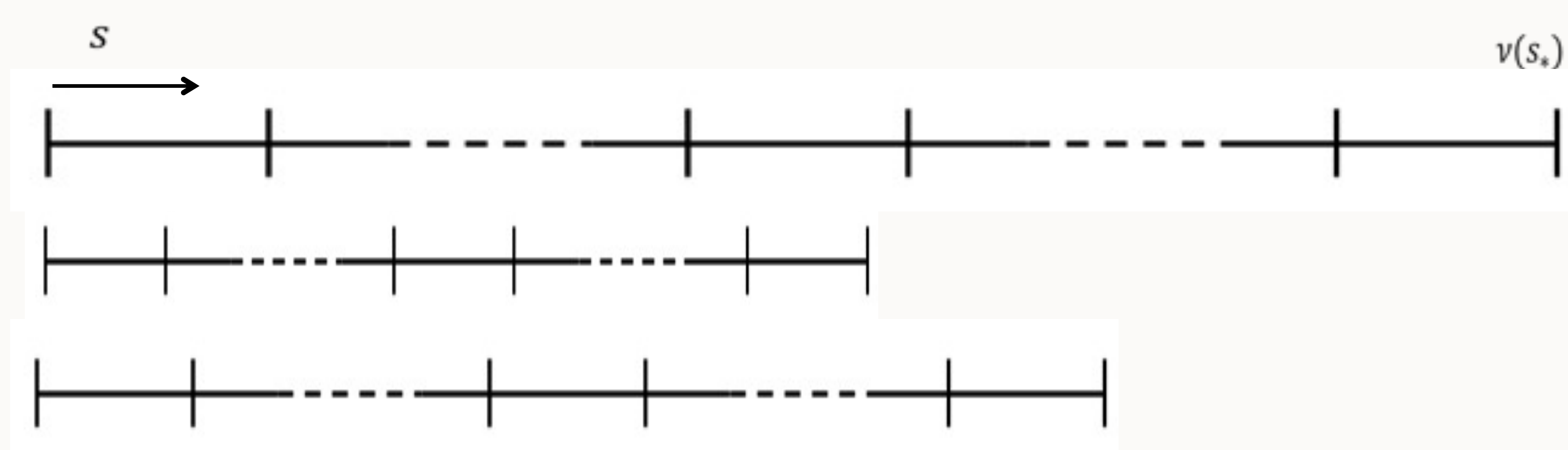
$$v(s) = \frac{v(z)}{v(L)}$$

Initial Value Problem

$$\frac{d}{ds}\begin{pmatrix} \varrho \\ \tau \\ \upsilon \end{pmatrix} = \begin{pmatrix} sign(p) - \frac{\varrho^2+\tau^2}{\upsilon^2} \\ A - B\frac{\varrho^2+\tau^2}{\upsilon^2} \\ -\frac{\varrho+B\tau}{\upsilon} \end{pmatrix}$$

with Initial Conditions

$$\upsilon(0) = 1, \varrho(0) = \tau(0) = 0$$



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Solving for original end points

$s : 0 \to s_*$ and solving for the $\varrho(s_*)$, $\tau(s_*)$ and $\upsilon(s_*)$

$$L = \frac{s_*}{\upsilon(s_*)\sqrt{|p|r}}$$

$$\upsilon(L) = \frac{1}{\upsilon(s_*)}$$

$$P(0) = \frac{\varrho(s_*)\sqrt{|p|/r}}{\upsilon(s_*)}$$

$$Q(0) = \frac{\tau(s_*)\sqrt{|p|/r}}{\upsilon(s_*)}$$

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MATLAB/RESULTS

INITIAL AND BOUNDARY VALUE PROBLEMS

IVP

$$-\frac{d}{ds}\begin{pmatrix} \rho \\ \tau \\ \upsilon \end{pmatrix} = \begin{pmatrix} \mathrm{sign}(p) - \frac{\rho^2+\tau^2}{\upsilon^2} \\ A - B\frac{\rho^2+\tau^2}{\upsilon^2} \\ -\frac{\rho+B\tau}{\upsilon} \end{pmatrix}$$

BVP

$$\frac{d}{dz}\begin{pmatrix} P \\ Q \\ v \end{pmatrix} = \begin{pmatrix} p - r\frac{P^2+Q^2}{v^2} \\ q - x\frac{P^2+Q^2}{v^2} \\ -\frac{rP+xQ}{v} \end{pmatrix}$$

Graphs generated from each problem:

End Voltage vs. Length

Power Utilization vs. Length

Voltage vs. Position

Power vs. Position

MATLAB CODE

- IVP
  - ode45 used to solve the system of rescaled ODEs
- BVP
  - bvp4c was used to solve the original system of ODEs
  - Requires a guess of the solution

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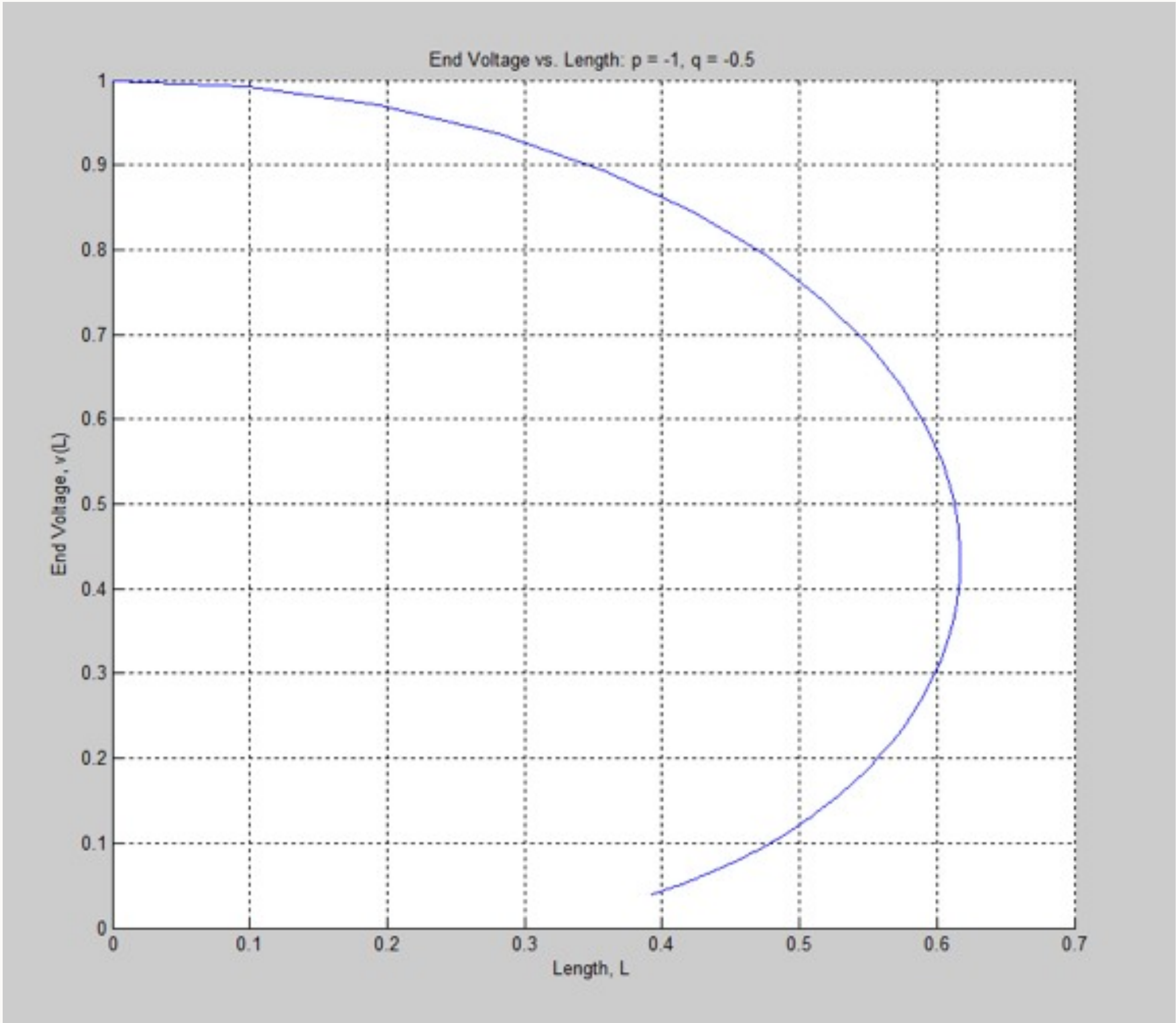
End Voltage vs. Length

p = -1, q = -0.5



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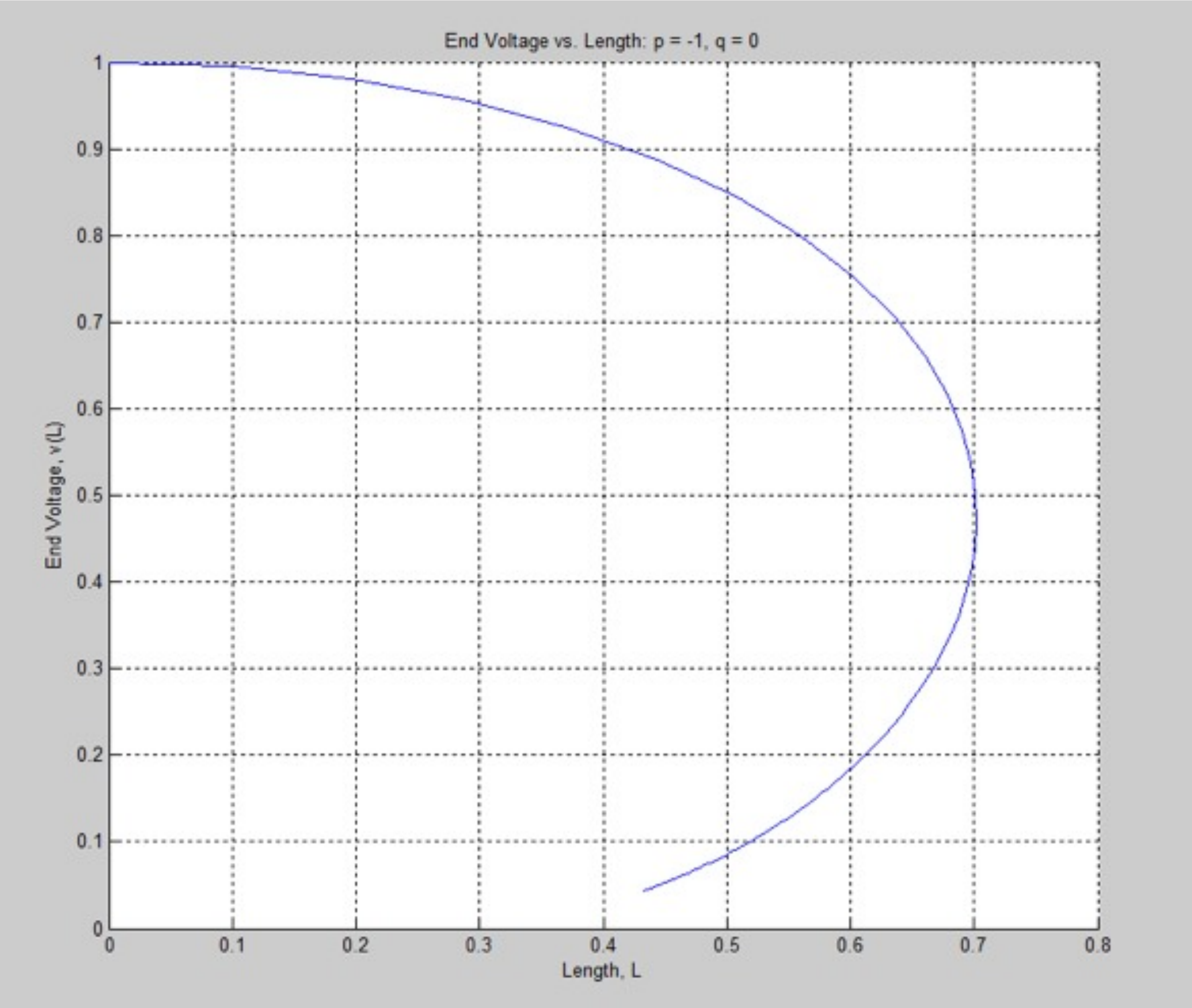
End Voltage vs. Length

p = -1, q = 0



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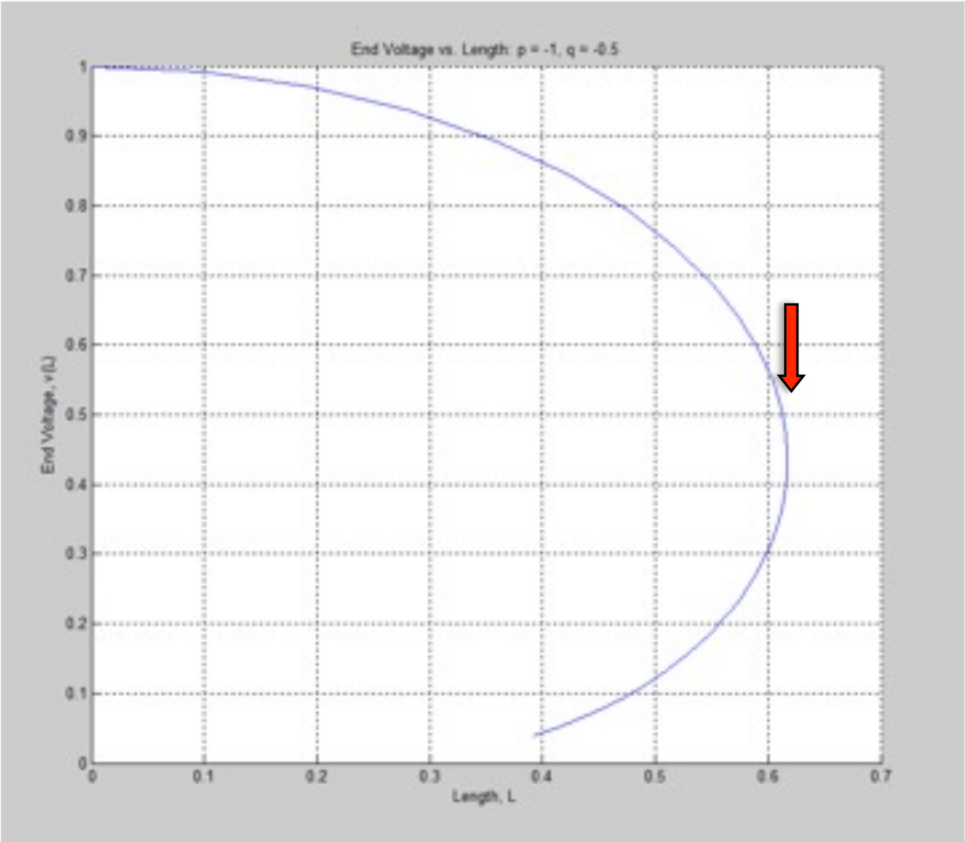
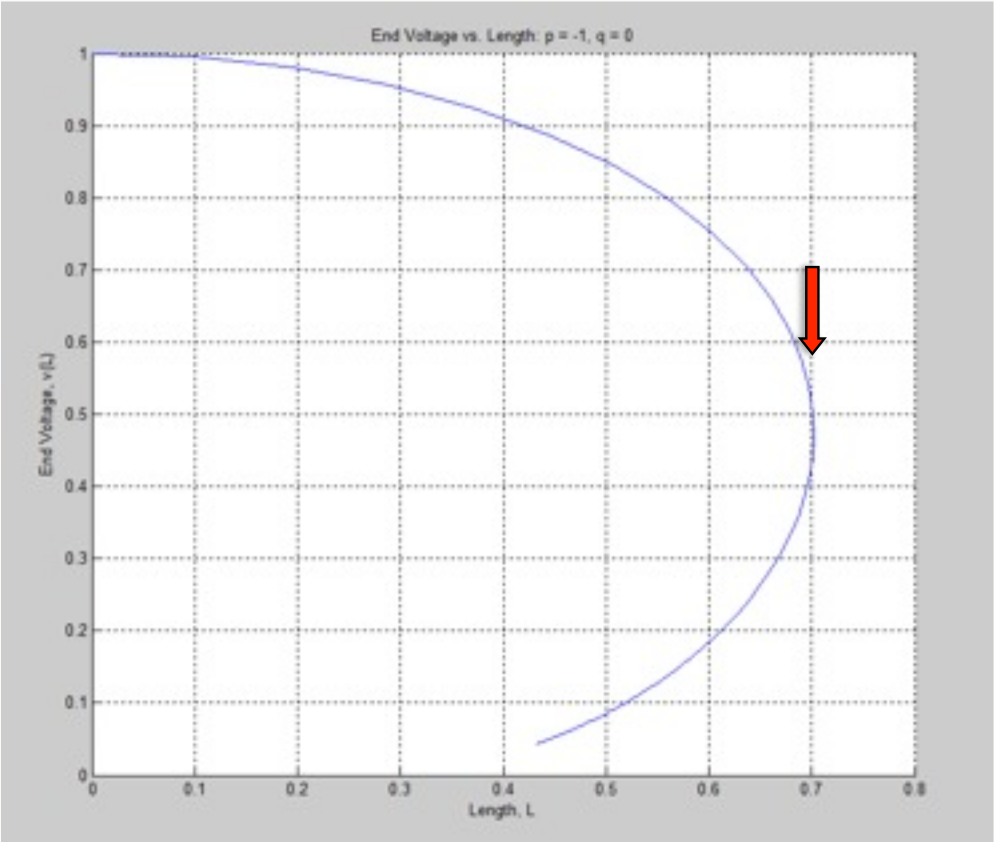
p = -1, q = -0.5



p = -1, q = 0



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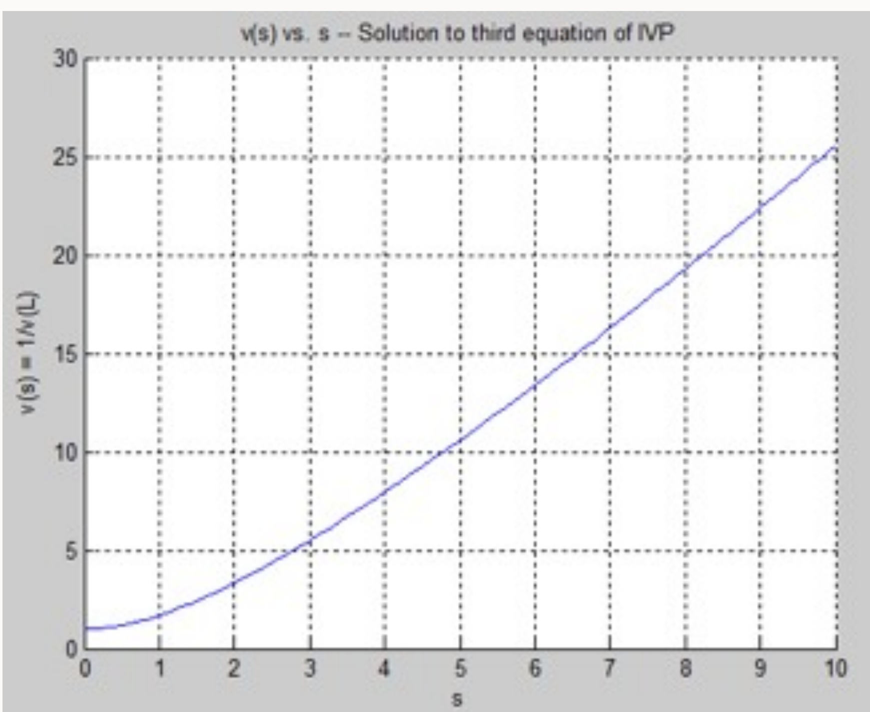
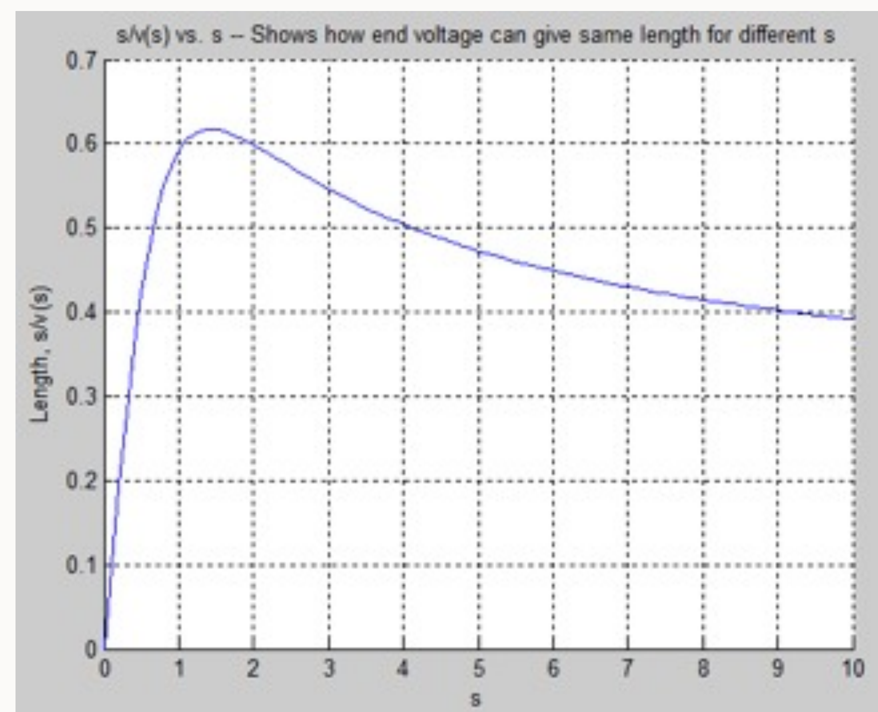
Two End Voltages for One Length

$$v(L) = \frac{1}{\upsilon(s_*)}$$

$$L = \frac{s_*}{\upsilon(s_*)\sqrt{|p|r}}$$





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POWER UTILIZATION VS. LENGTH

$$\text{Power utilization} = \frac{\text{Initial Injected Power}}{\text{Power Consumed}} = \frac{P(0)}{p*L}$$

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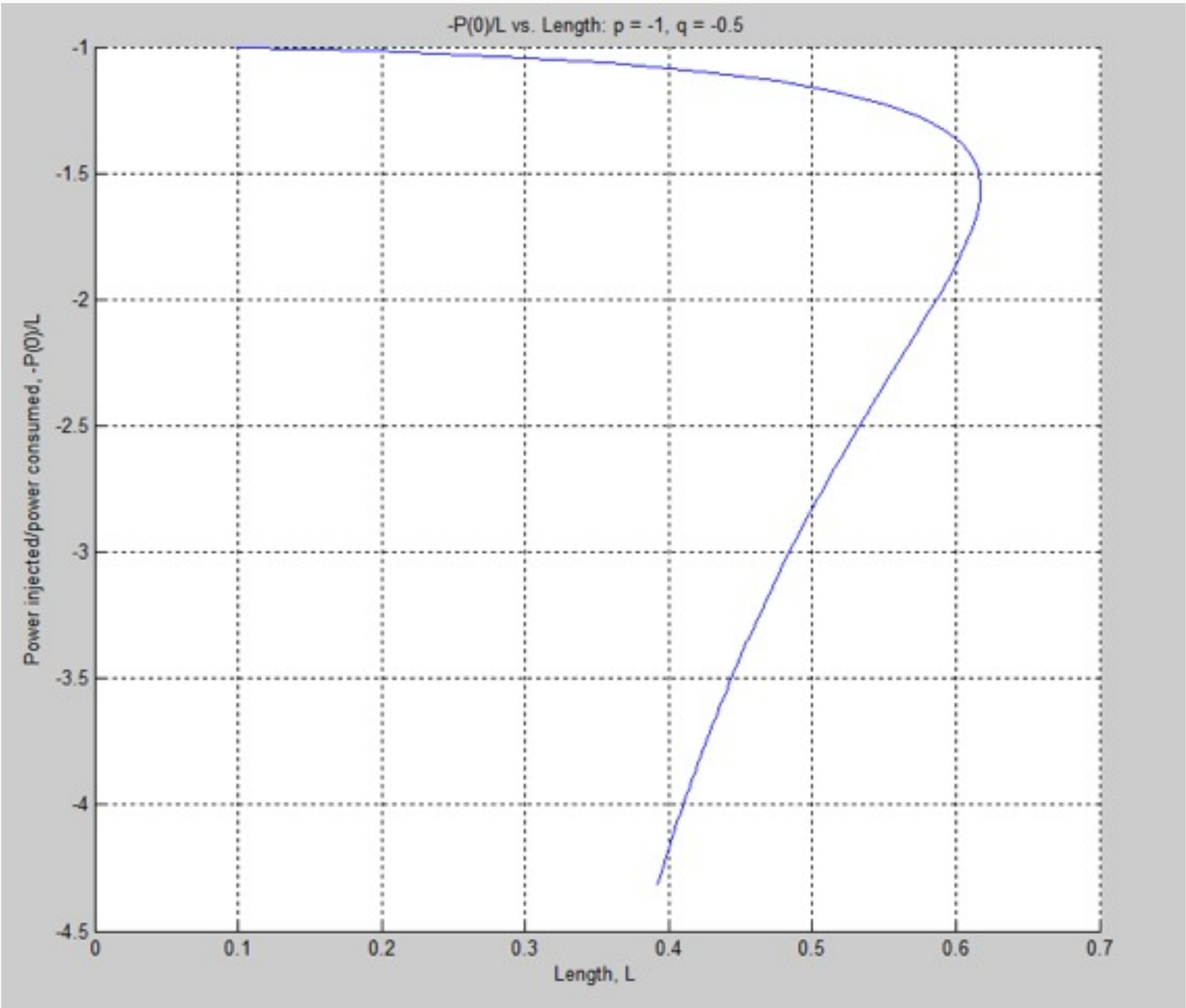
Power Utilization vs. Length
p = -1, q = -.5



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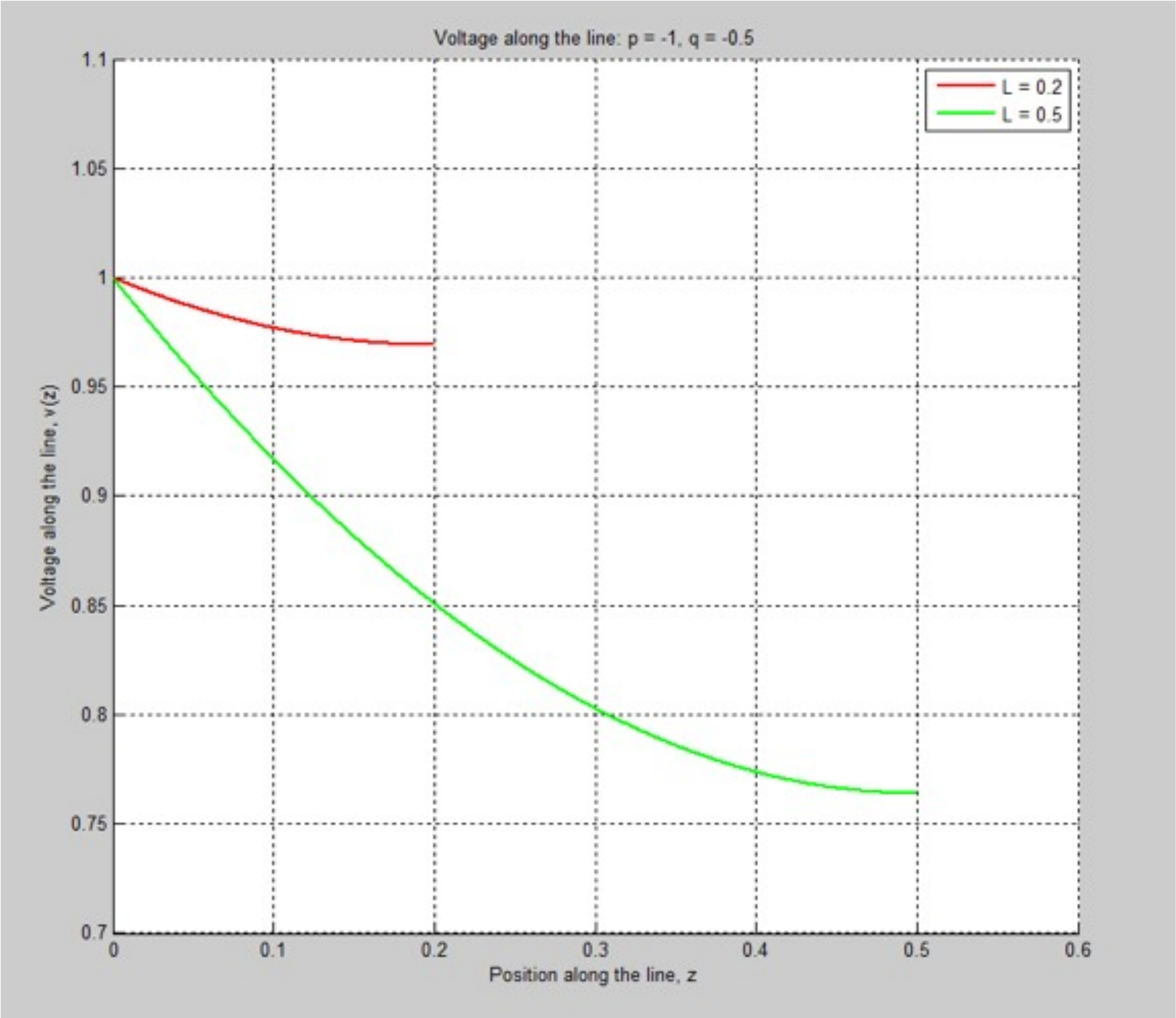
Voltage Along the Line



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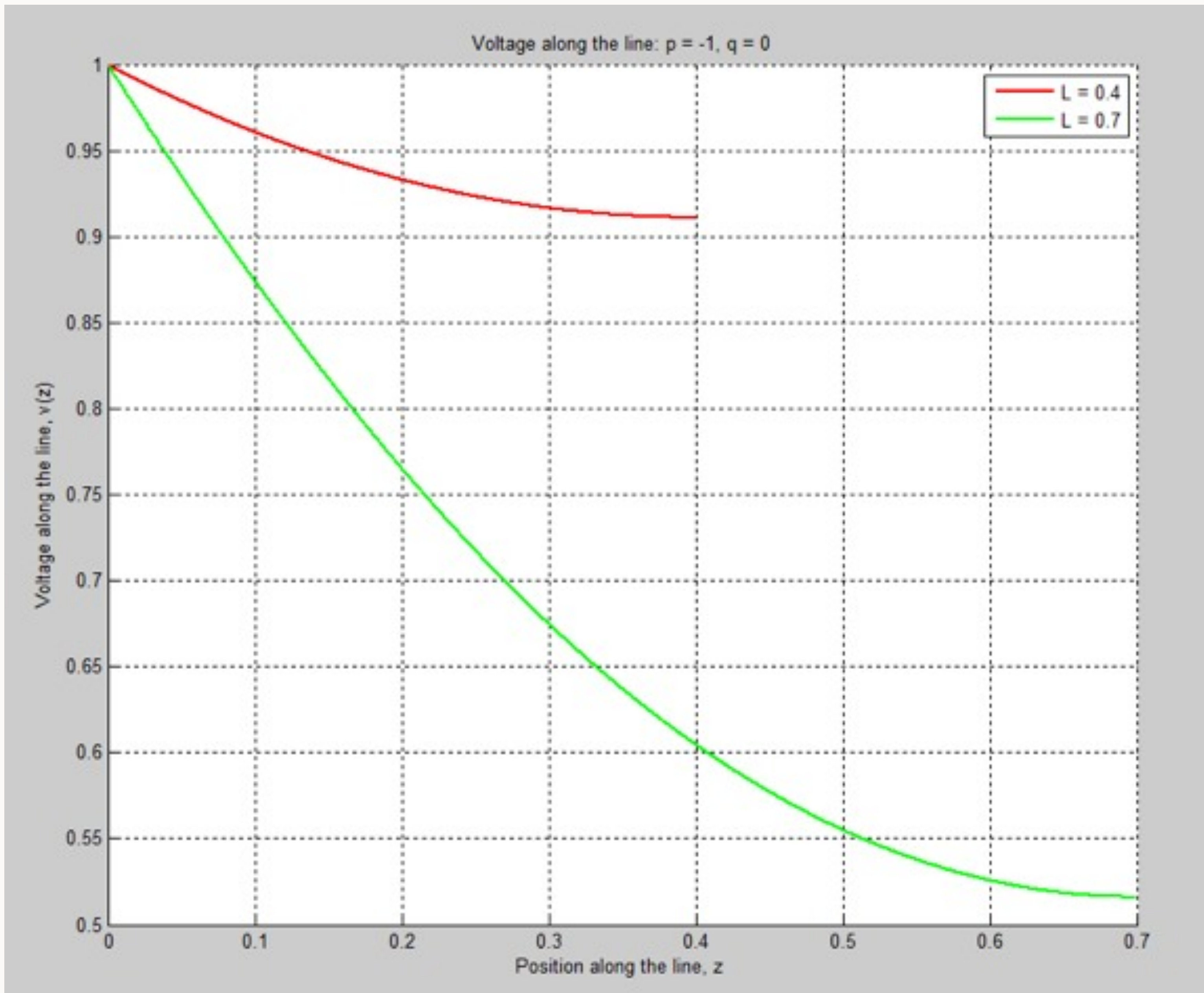
Voltage Along the Line



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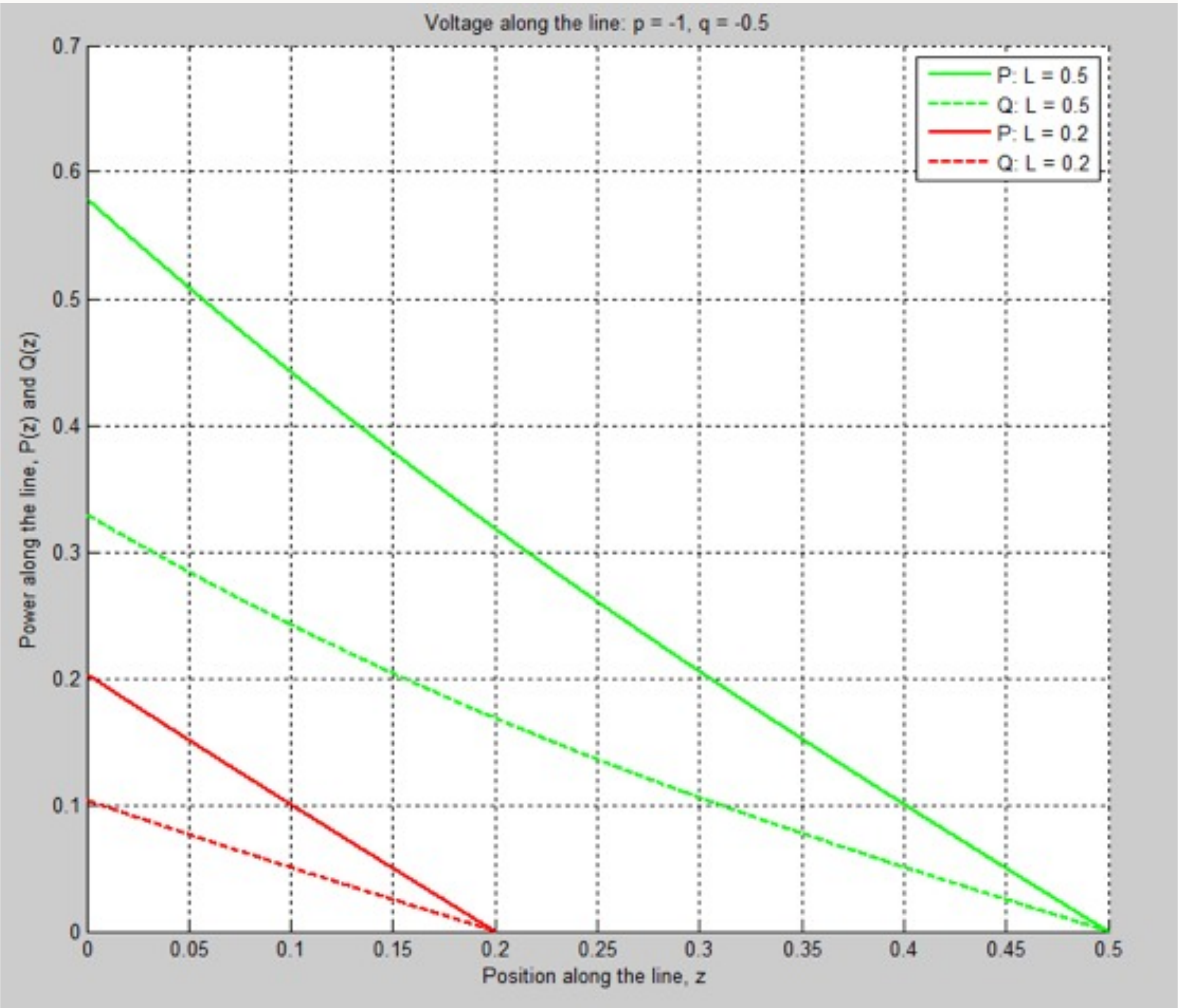
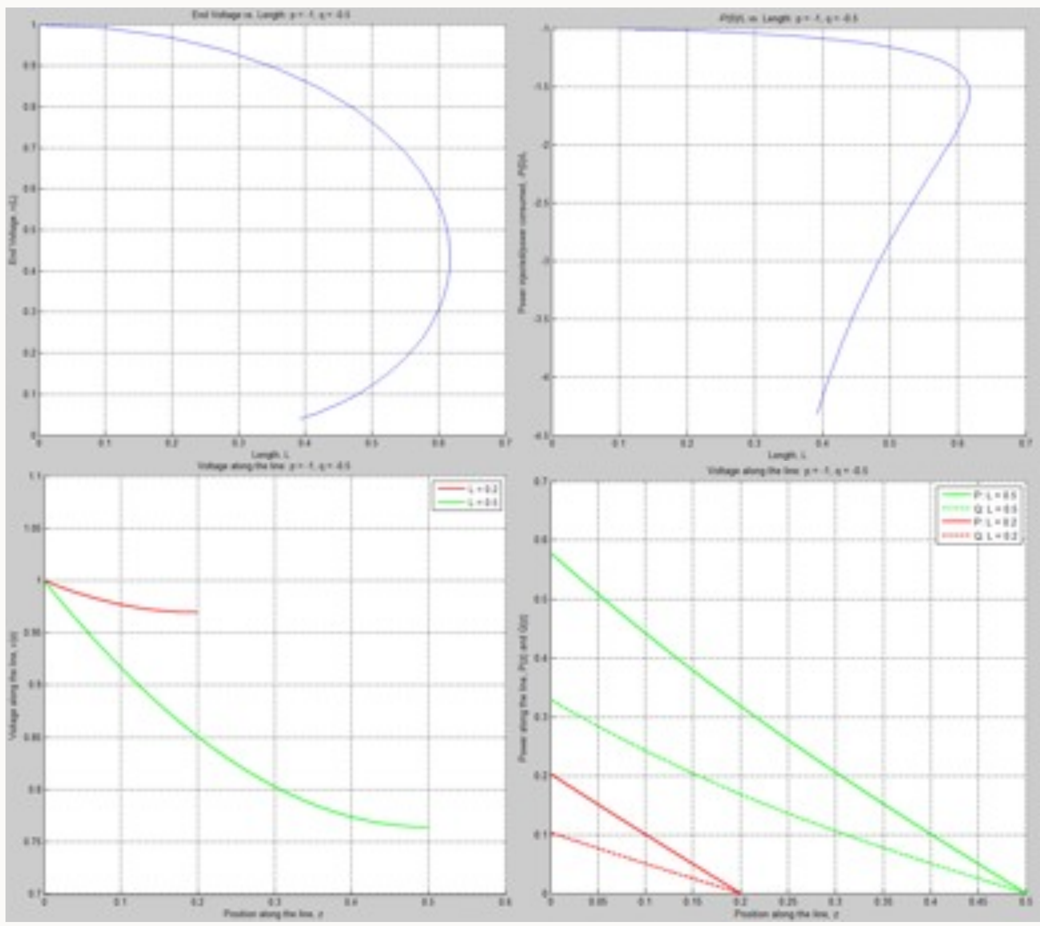
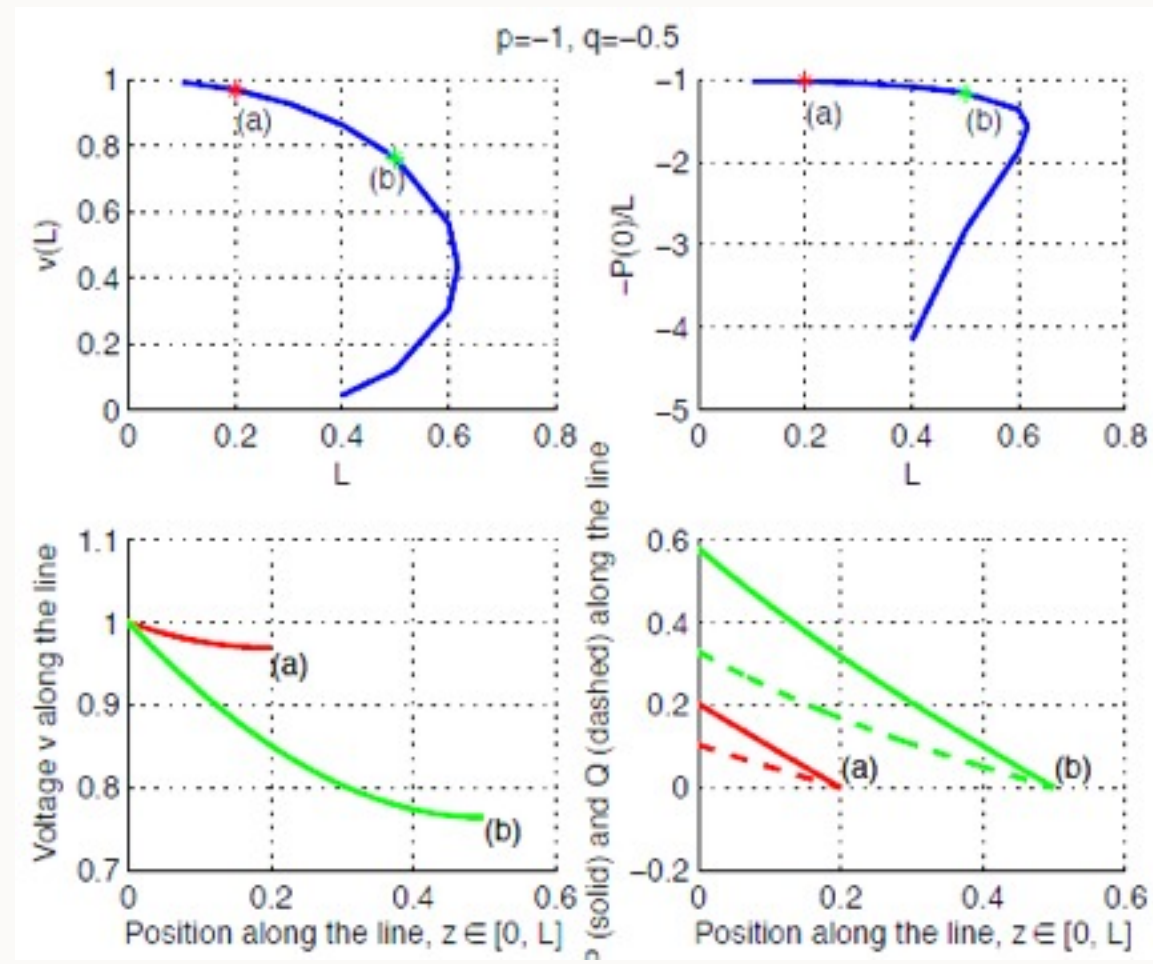
Power Along the Line



OUR GRAPHS VS. ARTICLE





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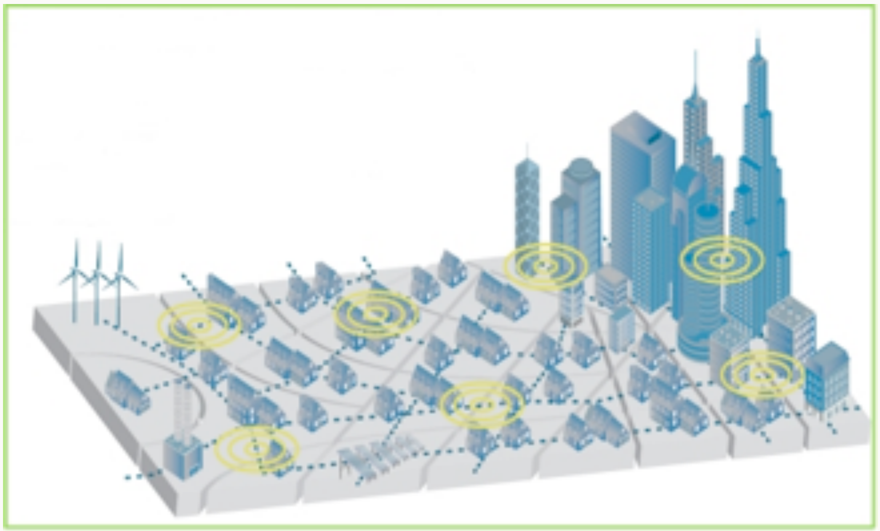
FUTURE WORK

FUTURE WORK

- Major Assumption:
  - Uniform consumption of loads *p*
- In reality
  - Slight variations across a line



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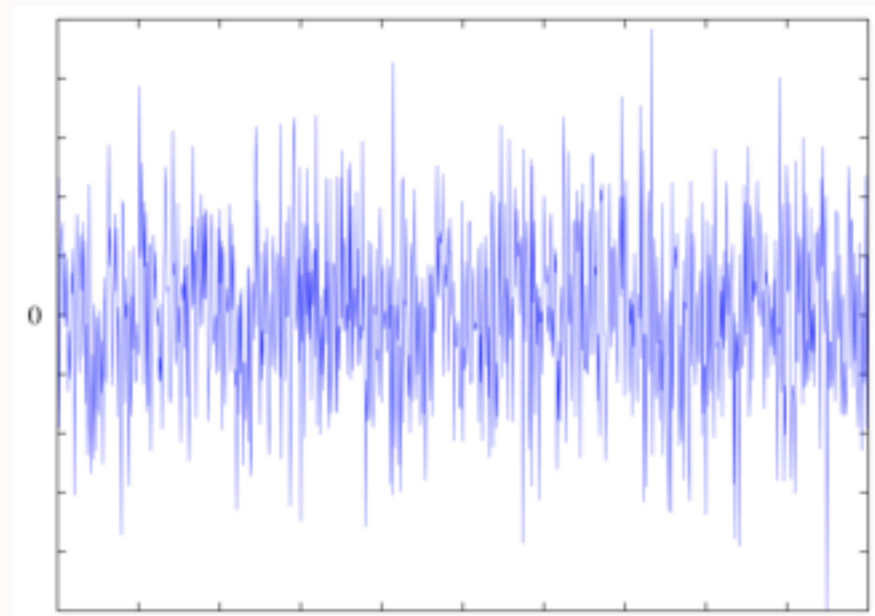
OUR PLAN



- Monte Carlo Method
  - Let $p(l) = p_0 + w(l)$
  - Substitute for p in DistFlow ODEs
  - Solve for boundary value problem
  - Repeat

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Expectations

- Verify the previous researcher's approach to statistical similarity as valid
- Averaged results will align with previous findings

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REFERENCES

- The papers used for this presentation were:

- M. Baran and F. Wu, "Optimal sizing of capacitors placed on a radial distribution system," *Power Delivery, IEEE Transactions on*, vol. 4, no. 1, pp. 735 –743, jan 1989.

- D Wang, K Turitsyn and M Chertkov, "DistFlow ODE: Modeling, Analyzing and Controlling Long Distribution Feeder", Proceedings of, the 51st IEEE Conference on Decision and Control (2012) [http://arxiv.org/abs/1209.5776]

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QUESTIONS?