SINKING BUBBLES IN AN OSCILLATING FLUID

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STATEMENT OF PROBLEM, PHENOMENON

- It is observed that bubbles can sink in a quickly oscillating container

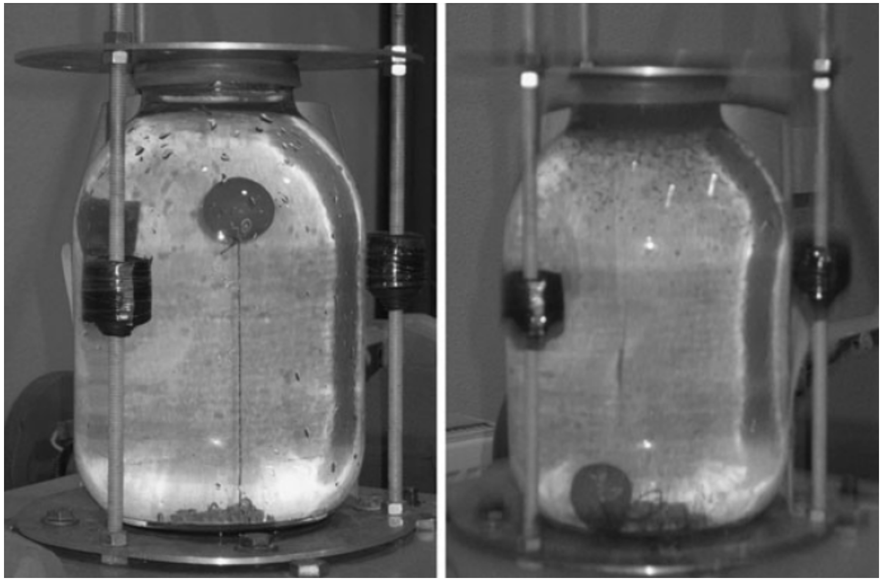
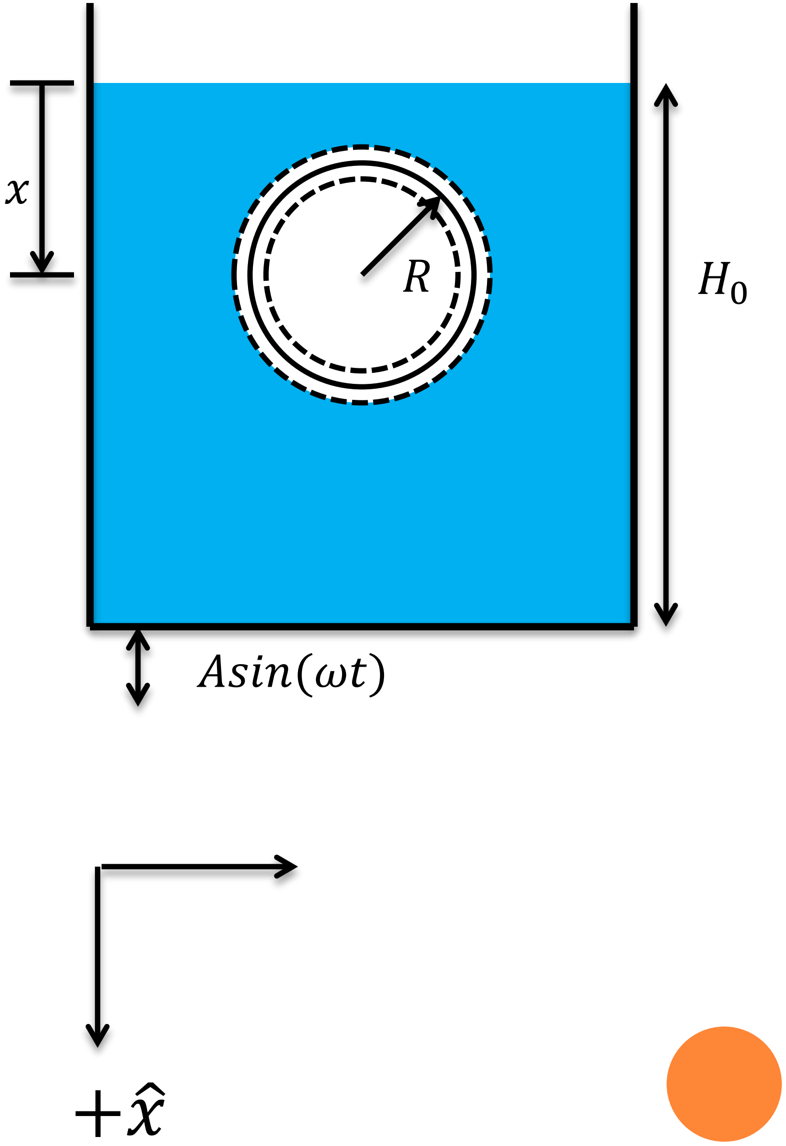


Figure credit [1]

- Why? Added mass.

PHYSICAL MODEL

- Variables and parameters
  - x – Bubble depth
  - R – Bubble radius
  - H_0 – Total height of water
  - A – Amplitude of oscillations
  - ω – Frequency of oscillations
  - t – Time

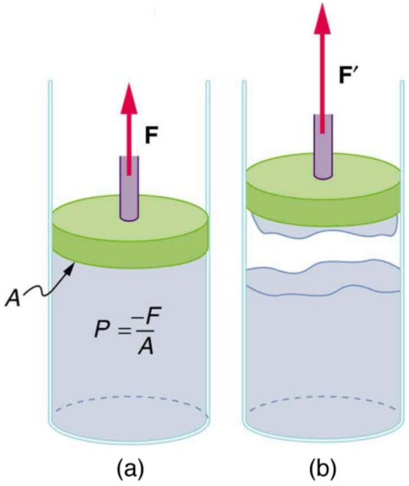


MODEL ASSUMPTIONS

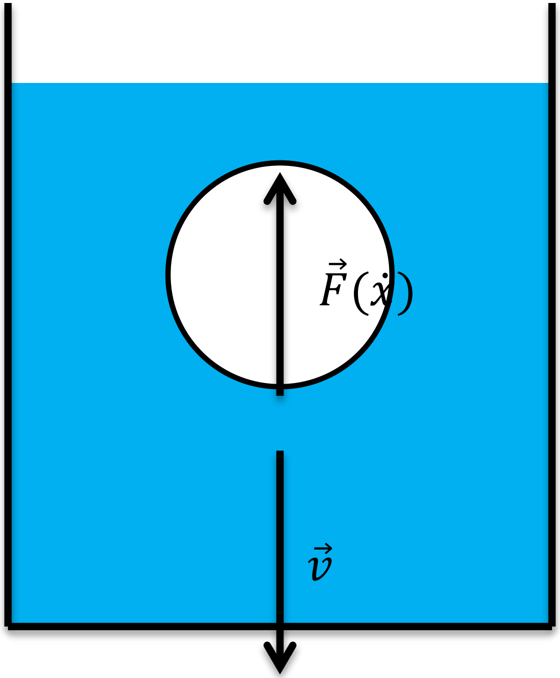
- Spherical bubble
    - Small bubble
    - Compressible
- Quasistatic
- External pressure >> Pressure in water
    - Prevents cavitation

- $V_b = \frac{4}{3}\pi R^3$
    - $R < 2cm$

- $P_x V_b = P_e V_{b0}$
- $P_e \gg \rho H_0 (g + A\omega^2)$

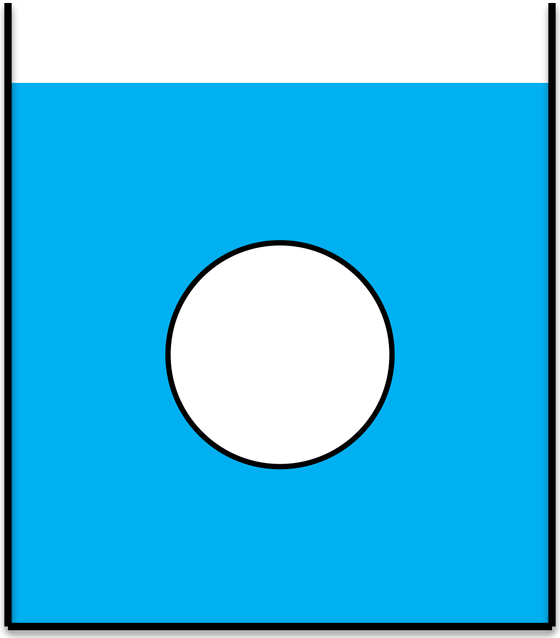


"FRICTIONAL" FORCES – DRAG



- Bubble has an instantaneous velocity
- The fluid resists the motion of the bubble
- $F(\dot{x}) = \frac{1}{2}\rho\dot{x}^2 C_d A$
    - C_d – drag coefficient
    - A – surface area

BUBBLE VOLUME



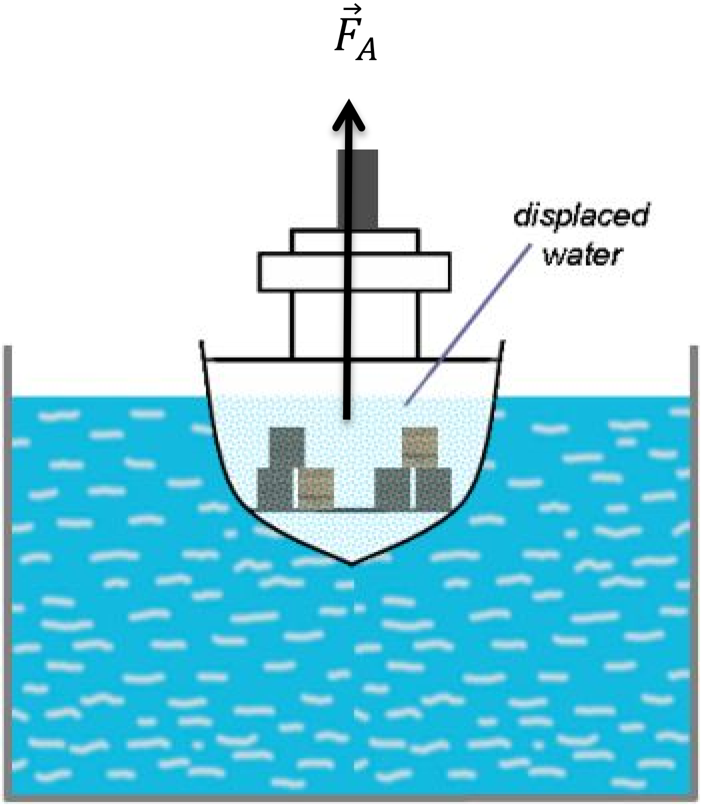
- Bubble doesn't move same distance as the tank
- Causes bubble depth to change
  - Bubble expands/shrinks

$$V_b = V_{b0}\left[1 - \frac{\rho x g}{P_e}\left(1 + \frac{A\omega^2}{g}\sin \omega t\right)\right]$$

ARCHIMEDES' FORCE

- Buoyant force
- Always upwards
- Why bubbles usually float

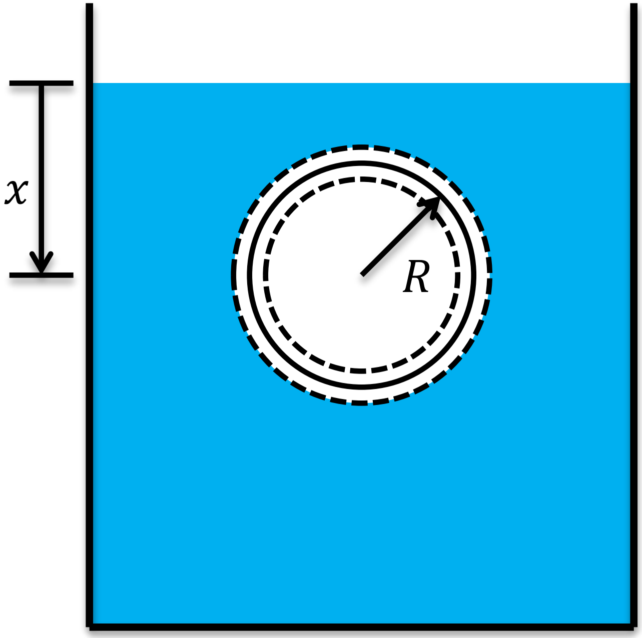
$$F_A = -\rho V_b g$$



PHENOMENON OF ADDED MASS

- Bubble has to displace the water around it to move

- Results in a larger observed inertia for bubble (bubble gained mass)



$$m_0 = \frac{1}{2}\rho V_b$$

m_0 changes with time!

$$M = m + m_0$$

added mass

Intrinsic mass of bubble

GOVERNING EQUATION – PART 1

- Newton's Second Law

$$\xcancel{\vec{F} = M\vec{a}}$$

- Only valid for constant mass (inertia)
- Added mass changes with bubble size
- More general:

$$\vec{F} = \frac{d\vec{P}}{dt} \quad ; \quad \vec{P} \equiv M\vec{v}$$

$$F = \frac{dP}{dt}$$

$$F = \frac{d((m + m_0)\dot{x})}{dt}$$

$$F = (m + m_0)\frac{d\dot{x}}{dt} + \dot{x}\frac{d(m + m_0)}{dt}$$

$$\boxed{F = (m + m_0)\ddot{x} + \dot{m}_0\dot{x}}$$

Added Mass

GOVERNING EQUATION – PART 2

- Plugging in all the forces we get:

$$(m+m_0)\ddot{x}+\dot{m}_0\dot{x}=-F(\dot{x})+(\mathrm{m}-\rho V_b)(\mathrm{A}\omega^2\sin\omega t+\mathrm{g})$$

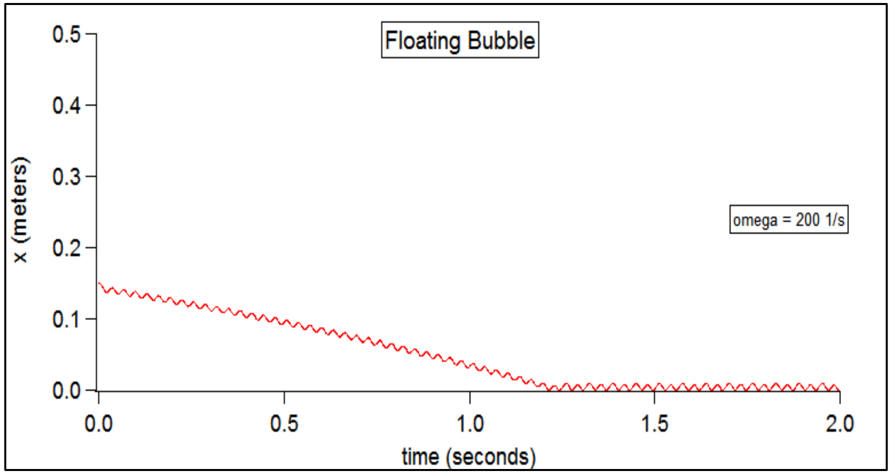
- To get equation of motion, plug in:

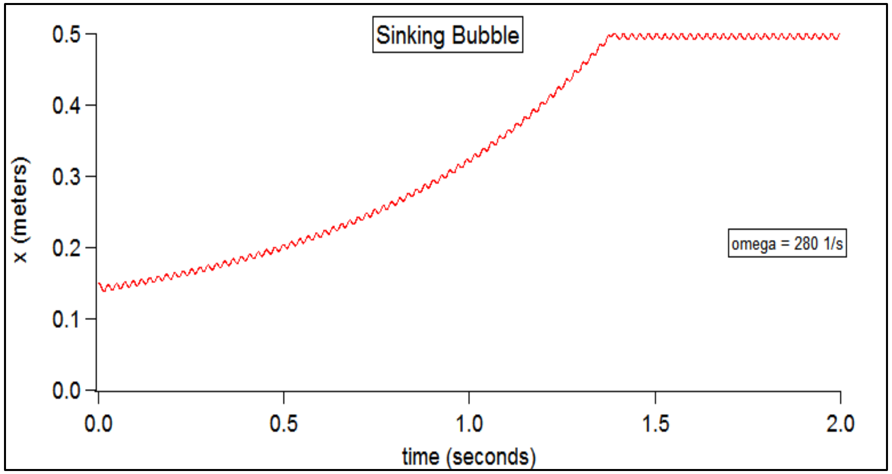
$$m_0=\frac{1}{2}\rho V_b$$

$$F(\dot{x})=\frac{1}{2}\rho\dot{x}^2C_dA*\mathrm{sign}(\dot{x})$$

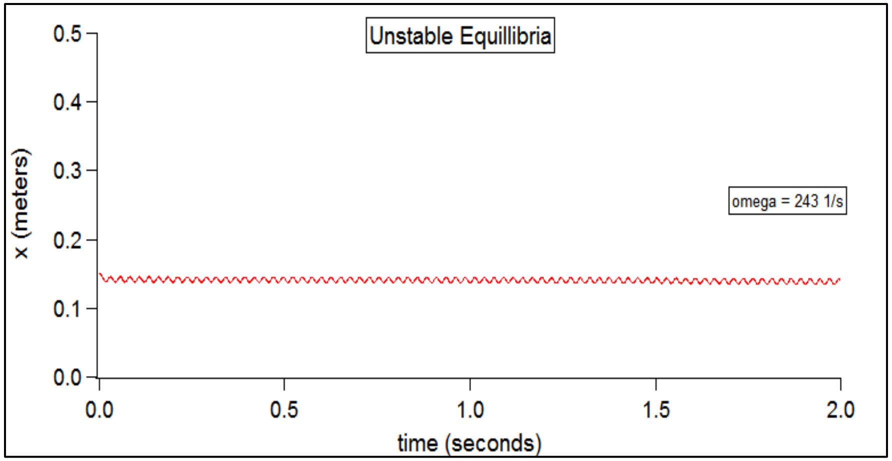
$$V_b{=}V_{b0}\left[1-\frac{\rho xg}{P_e}\left(1+\frac{A\omega^2}{g}\sin\omega t\right)\right]$$

RESULTS BY DIRECT INTEGRATION

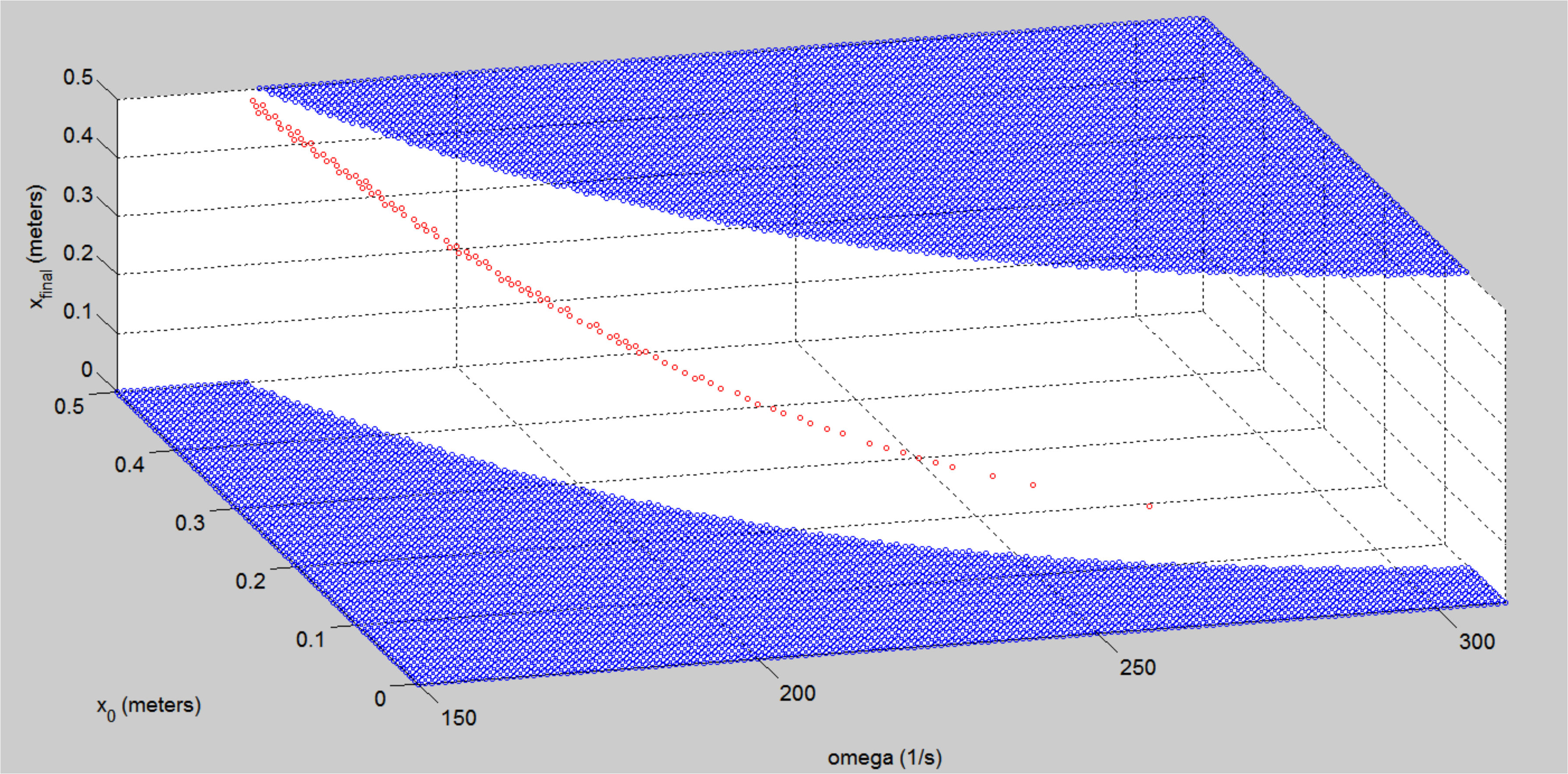




- Parameters
  - x_0 – 0.15 m
  - H_0 – 0.5 m
  - A – 0.002 m
  - R_0 – 0.0029 m
  - ρ_{fluid} – 1000 kg/m^3
  - ρ_{bubble} – 1.2 kg/m^3
  - P_e – 10^5 Pa



BIFURCATION DIAGRAM



● -- Stable

● -- Unstable

CONCLUSION

- Bubbles in oscillating fluids sometimes sink
- Extremely dependent on:
  - Height of the bubble
  - Frequency of oscillations

- Why?
- Added mass term beats out buoyancy term
- Causes average position of bubble to decrease
- Friction slows processes in either direction but not required for sinking to occur

WHAT TO DO NEXT

- Use vibrational mechanics to time average equation of motion
- Consider bubble initial size as a parameter
- Add effects of container elasticity
- Model for multiple bubbles
  - Bubble cluster behavior

SOURCES

[1] Sorokin, V. S., Blekhman, I. I., Vasilkov, V. B.: Motion of a gas bubble in fluid under vibration. Nonlinear Dyn (2012) 67:147–158.

[2] http://web.mit.edu/2.016/www/handouts/Added_Mass_Derivation_050916.pdf

QUESTIONS?

- Special thanks to Dr. Gabitov and Matt Pennybacker