# Math 322. Spring 2008
# Review Problems for Mid Term 1

## Chapter 13 (Complex Numbers):

**Topic 1: Polar form of complex number .**

**Question 1.**

Let $z = 1 - i$. Evaluate $w = 1/z$ in polar form, with the principal argument.

**Question 2.**

Let $z_1 = -2 + 2i$ and $z_2 = -6 - 6i$. Evaluate $Arg(z_1/z_2)$.

**Topic 2: Operations of complex numbers.**

**Question 3.**

Let $z_1 = 3 + 2i$, $z_2 = 2 - 2i$, find

$$\text{(a)} \quad \frac{z_1 + z_2}{\bar{z}_2^2} \qquad \text{(b)} \quad \text{Im}([(1-i)^8 z_1^2])$$

$$\text{(c)} \quad |\frac{z_1 - z_2}{z_2}| \qquad \text{(d)} \quad \text{Re}((z_1 + 1)z_2)$$

**Topic 3: Roots of complex number .**

**Question 4.**

Find all the solutions for $z^4 = 1$.

**Question 5.**

Find all the solutions for $z^3 = 2 - 2i$.

**Topic 4: Continuity, Differentiability and analyticity of complex functions, Cauchy-Riemann equations, harmonic function and harmonic conjugate**

**Question 6.** Find out whether the following function is continuous at $z = 0$.

$$f(z) = \begin{cases} \frac{\text{Im}(z^2)}{|z|^2}, & z \neq 0; \\ 0, & z = 0. \end{cases}$$

**Question 7.** Use the Cauchy-Riemann equations to check whether the following function is analytic.

$$e^{-x}(\cos y - i \sin y)$$

**Question 8.** Use the Cauchy-Riemann equations to check whether the following functions are analytic.

$$
\begin{aligned}
&(a) \quad f(z) = z + \bar{z} \\
&(b) \quad g(z) = 3z - 2\bar{z} \\
&(c) \quad h(z) = \frac{\bar{z}}{|z|^2}
\end{aligned}
$$

**Question 9.**

Is the following function analytic when $z \neq 0$?

$$f(z) = i/z^2$$

**Question 10.**

If $f(z) = f(x + iy) = u(x, y) + iv(x, y)$ is an analytic function, prove that $v = v(x, y)$ is a harmonic function.

**Question 11.**

Verify that $u(x, y) = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a harmonic conjugate function $v(x, y)$ of $u(x, y)$.

**Question 12.**

Find $a$ such that the function $u(x, y) = e^{3x} \cos ay$ is harmonic.

**Question 13.**

Determine $c$ such that the function $u(x, y) = \sin x \cosh cy$ is harmonic. Find a harmonic conjugate.

**Topic 5: Exponential, trigonometric, hyperbolic and logarithmic functions, general power.**

**Question 14.**

Let $z = x + iy$. Find the Re and Im of $e^{1/z}$.

**Question 15.**

Find the Re, Im and modulus of $e^{-3+\frac{4\pi}{7}i}$.

**Question 16.**

Compute $\sin(5 - 2i)$.

**Question 17.**

Compute $\cosh((n + \frac{1}{2})\pi i)$, where $n$ is an integer.

**Question 18.**

Show the following identity is true. (Hint: You may need to use the identity $e^{inx} = (e^{ix})^n$).

$$\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta)$$

**Question 19.**

Compute $\mathrm{Ln}(5 - 4i)$, $\mathrm{Ln}(-2)$.

**Question 20.**

Find the principal value of $(1+i)^{1-i}$.

## Chapter 7 (Linear Algebra):

### Topic 1: Matrix Operations.

**Question 1.**

Which of the following equations may not be true? Why not?
$(a)$ $A(BC) = (AB)C$
$(b)$ $(A+B)C = AC + BC$
$(c)$ $(A+B)^2 = A^2 + 2AB + B^2$
$(d)$ $(AB)^T = B^T A^T$

**Question 2.**

Let

$$A=\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 0 & 5 \end{bmatrix}, \quad B=\begin{bmatrix} 3 & 2 & 1 \\ 4 & -2 & 3 \end{bmatrix}$$

Calculate the following products or sums or give reasons why they are not defined.
(a) $AB$ (b) $BA$ (c) $A+B$ (d) $A-B^T$

### Topic 2: Linear Independence of vectors .

**Question 3.**

Explain why if A is not square, then either the row vectors or the column vectors of A are linearly dependent.

**Question 4.**

Are the following vectors linearly independent: [1 2 3 4], [2 3 4 5], [3 4 5 6], [4 5 6 7]?

**Question 5.**

Are the following vectors linearly independent: [3 4 7], [2 0 3], [8 2 3], [5 5 6]?

### Topic 3: Null space.

**Question 6.**

Find the null space of the following matrix:

$$A = \begin{pmatrix} 10 & 1 & 2 \\ 0 & 1 & 6 \end{pmatrix}.$$

**Question 7.**

Find the null space of the following matrix:

$$A = \begin{pmatrix} 10 & 1 & 2 & 3 \\ 0 & 1 & 6 & 8 \end{pmatrix}.$$

**Topic 4: Linear system of equations, rank, row space, column space, basis**

**Question 8.**

Find the rank, a basis for the row space, and a basis for the column space of the following matrix.

$$\begin{pmatrix} 8 & 2 & 5 \\ 16 & 6 & 29 \\ 4 & 0 & -7 \end{pmatrix}$$

**Question 9.**

Let

$$A = \begin{pmatrix} -2 & 2 & 6 \\ 1 & -1 & 2 \\ -1 & 1 & 3 \end{pmatrix}.$$

Does the system $Ax = B$ with $B = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ admit solutions? If so, how many?

**Question 10.**

Does rank $A$ = rank $B$ imply rank $A^2$ = rank $B^2$? If yes, justify your answer; otherwise, give a counterexample.

**Question 11.**

Let

$$\text{A}= \begin{bmatrix} 0 & -6 & 4 \\ 1 & -2 & -2 \\ 1 & -8 & 2 \\ 3 & -12 & -2 \end{bmatrix}$$

(a) Find the rank of $A$.
(b) Find a basis of the column space of $A$.
(c) Find a basis of the row space of $A$.

(d) Let $x = [x_1, x_2, x_3]^T$. Find the general form of solutions for the homogeneous linear system of equations

$$AX = 0.$$

(e) Find the dimension of the null space of $A$.

(f) Let $b = [1, 2, 3, 7]^T$. Does the following system of equations have solution(s)? If your answer is yes, find the general form of the solution(s).

$$AX = b.$$