Math 322. Spring 2008.
Review problems for midterm 2

Part one: linear algebra

Topic: Determinant.
Problem 1: Find the determinant of the following matrix.

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 3 \\ -1 & 1 & 0 \end{pmatrix}.$$

Problem 2: Let $A = \begin{pmatrix} a & b & 0 & 0 \\ b & a & 0 & 0 \\ 0 & 0 & c & d \\ 0 & 0 & d & c \end{pmatrix}$. Find $\det(A)$.

Topic: Inverse.
Problem 3: Find the inverse of the following matrix, or argue why it doesn't exist.

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 0 & 0 \\ -1 & 2 & 1 \end{pmatrix}.$$

Problem 4: Let $A = \begin{pmatrix} -1 & 1 & 3 \\ 2 & 8 & 9 \\ 3 & 7 & 6 \end{pmatrix}$. Find A^{-1}, or argue why it doesn't exist.

Topic: Eigenvalues and eigenvectors .

Problem 5:

Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$

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Problem 6:

Find the eigenvalues and eigenvectors of

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

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Part two: Ordinary differential equations

Topic: Linear independency of functions)
Problem 1:

Find an ODE for which the given functions $y_1 = \cos \omega x$ and $y_2 = \sin \omega x$ are solutions. Verify these two functions are independent.

Problem 2:

Find an ODE for which the given functions $y_1 = e^x$ and $y_2 = xe^x$ are solutions. Verify these two functions are independent.

Topic: Linear ODE (Existence and uniqueness of solutions, solving homogeneous and inhomogeneous linear ODE)

Problem 3: Find the general form of solution to the following equation.

$$\frac{d^3y}{dx^3} - \frac{dy}{dx} = 0$$

Problem 4: Consider the following initial value problem.

$$\begin{aligned} \frac{d^2y}{dx^2} - 4y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{aligned}$$

(a). Does this initial value problem have a solution? Is the solution unique?
(b). Find the solution to this initial value problem if your answer for part (a) is yes.

Problem 5: Find the general form of solution to the following equation.

$$\frac{d^2y}{dx^2} + 5y = \cos x$$

Problem 6: Consider the following **inhomogeneous** initial value problem.

$$\begin{aligned} \frac{d^2y}{dx^2} - 4y = xe^{2x} \\ y(0) = 0 \\ y'(0) = 0 \end{aligned}$$

Find the general form of solution.

Problem 7: Solve the following **inhomogeneous** initial value problem.

$$\begin{aligned} \frac{d^2y}{dx^2} + y = 2\sin(x) \\ y(0) = 1 \\ y'(0) = 0 \end{aligned}$$

Topic: Linear ODE system (Existence and uniqueness of solutions, solving homogeneous linear ODE system)

Problem 8: Consider the following initial value problem.

$$\begin{aligned}\frac{dy_1}{dx} &= y_1 + 2y_2 \\ \frac{dy_2}{dx} &= 5y_1 - 2y_2 \\ y_1(0) &= 2 \\ y_2(0) &= 9\end{aligned}$$

(a). Does this initial value problem have a solution? Is the solution unique?
(b). Find the solution to this initial value problem if your answer for part (a) is yes.

Problem 9: Let $A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & -2 & 3 \\ 0 & 5 & -4 \end{pmatrix}$ and $\vec{Y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$. Find the general form of solution to the following system of equations.

$$\frac{d\vec{Y}}{dt} = A\vec{Y}$$

Problem 10: Consider again the ODE given in problem 2.
(a). Convert this problem into an first-order ODE system.
(b). Solve this ODE system, and compare the solution to the solution you found for problem 2.

Topic: Power Series
Problem 11: Find the radius of convergence of the following series:

$$\sum_{m=0}^{\infty} m!x^m.$$

Problem 12: Find the radius of convergence of the following series:

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{8^m} x^{3m}.$$

Problem 13:

Solve the following ODE by power series.

$$y' = 2xy$$

Problem 14:

Solve the following ODE by power series.

$$y'' + y = 0$$

Problem 15:

Solve the following ODE by power series.

$$y' = y + x$$

Topic: Sturm-Liouville equations

Solve the following Sturm-Liouville problem:

$$y'' + \lambda y = 0,\ y(0) = 0,\ y(5) = 0$$

Problem 16:

Solve the following Sturm-Liouville problem:

$$y'' + \lambda y = 0,\ y(0) = 0,\ y'(L) = 0$$

Problem 17:

Solve the following Sturm-Liouville problem:

$$y'' + \lambda y = 0,\ y(0) = y(2\pi),\ y'(0) = y'(2\pi)$$