Math 322. Spring 2015
Review Problems for Midterm 3

Fourier Series:

Topic: Calculation of Fourier coefficients

Question 1.

Find the Fourier series of the following periodic function, of period $p = 2L = 2\pi$:

$$f(x) = 1 - 2\sin^2(2x).$$

Hint: use the fact that $\cos 2x = \cos^2 x - \sin^2 x$ and $\cos^2 x = 1 - \sin^2 x$.

Question 2.

Find the Fourier series of the periodic function, $p = 2L = 2$,

$$f(x) = \left\{ \begin{array}{cc} 0 & -1 < x < 0 \\ x & 0 < x < 1 \end{array} \right\}.$$

Topic: Convergence of the Fourier series

Question 3.

For questions 1 and 2, describe what function the Fourier series represents on the whole real line.

Partial differential equations:

Topic: Sturm-Liouville problems

Question 1. For the following Sturm-Liouville problem, find the eigenvalues and eigenfunctions

$$y'' + \lambda y = 0, \quad y(0) = 0, \quad y'(L) = 0$$

Topic: Wave equation

Question 2: For a string of length π, with endpoints fixed, the eigenfunctions obtained by separation of variables are given by

$$u_n(x,t) = G_n(t)F_n(x) \ = (A_n \cos(nct) + B_n \sin(nct)) \sin(nx).$$

If the initial deflection is zero, i.e. $f(x) = 0$, and the initial velocity is given by $g(x) = \sin(x)\cos(x)$, find the deflection $u(x,t)$. Hint: Use the fact that $\sin 2x = 2\sin x \cos x$. **Explain your work in detail**.

Topic: Heat equation

Question 3: A metal bar of length $L = \pi$ is perfectly insulated, including being perfectly insulated at the endpoints (the temperature is not fixed there). It turns out that the situation of no heat flux through the ends corresponds to the conditions $u_x(0,t) = 0$, $u_x(\pi,t) = 0$. Find all eigenfunctions for the heat equation in this case. Explain your steps in detail. **Note that you do not need to match initial conditions for this problem!**