

TEST 1

October 8th, 2008

Your Name: _____

Directions:

- a. You may NOT use your book or your notes or a calculator.
- b. Please ask for extra scrap paper if needed.
- c. Show all work. Unless otherwise noted, a solution without work is worth nothing.
- d. Circle your answers.
- e. Good Luck!

Score:

1. _____

2. _____

3. _____

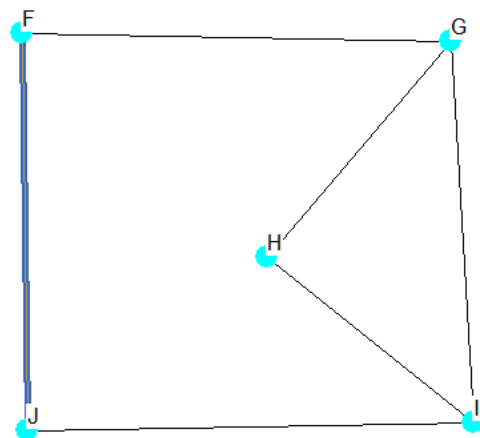
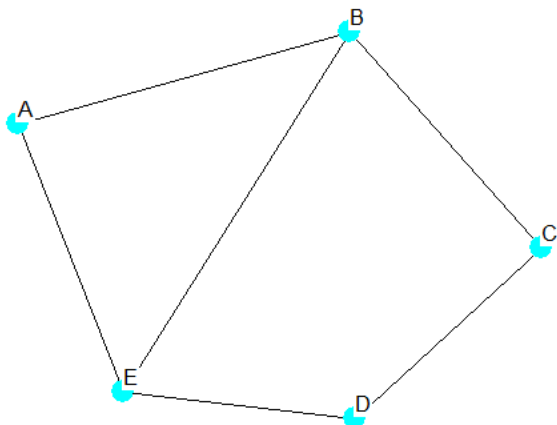
4. _____

5. _____

6. _____

Total _____

1. (16pts) Consider the following two graphs.



Suppose ϕ is an isomorphism between the graph on the left and the graph on the right. For the following statements, say if they are possible, impossible, or there is not enough information. (Note, do not assume a is true when checking b, etc.)

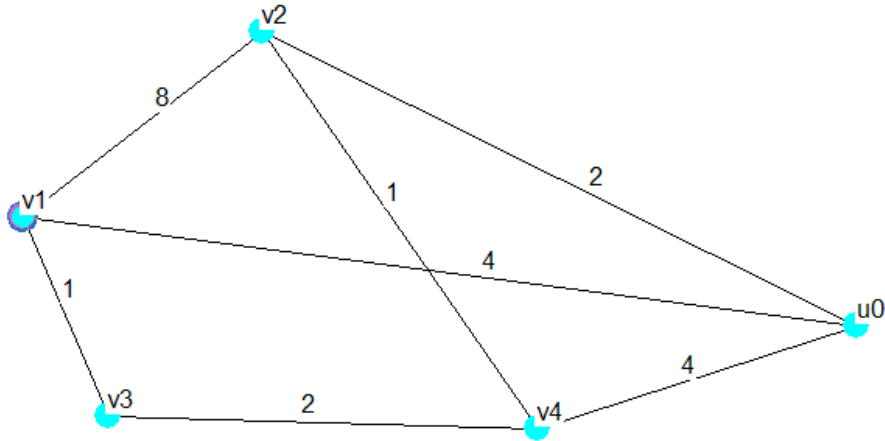
- a. $\phi(B) = F$
- b. $\phi(B) = G$
- c. $\phi(B) = H$
- d. $\phi(A) = \phi(B)$
- e. $\phi(C) = \phi(D)$
- f. $\phi^{-1}(H) = A$
- g. $\phi(BE) = GH$
- h. $\phi(AB) = GH$

2. (20pts)

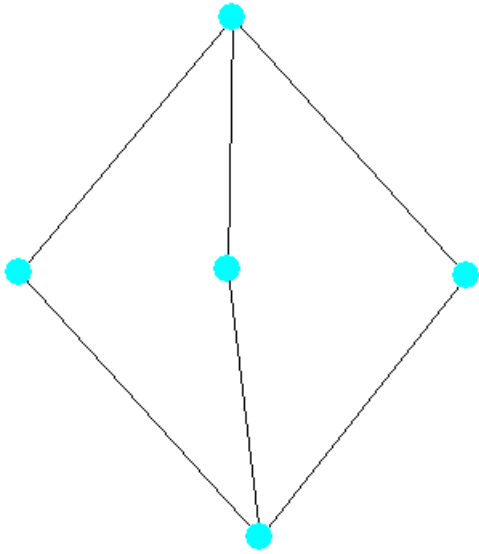
a. Give an example of a graph which is hamiltonian but not eulerian. Justify your answer.

b. Give an example of a graph which is eulerian but not hamiltonian. Justify your answer

3. (20pts) Find the distance between u_0 and all of the other vertices in this graph using Dijkstra's algorithm.



4. (15pts) Show that the graph G below is orientable. Assign appropriate directions to the edges of G to obtain a strongly connected digraph D .



5. (15pts) Suppose G is a tree with p vertices. What is the maximal degree of a vertex?

6. (15 pts) A plane triangulation is a plane graph such that every region is bounded by exactly three edges (including the exterior region). Show that if G is a plane triangulation with p vertices and q edges, then $q = 3p - 6$. (Hint: for each region add up the number of edges in its boundary and use Euler's theorem).