



Towards Improving the Integration of Undergraduate Biology and Mathematics Education

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Arguments have recently asserted the need for change in undergraduate biology education, particularly with regard to the role of mathematics. The crux of these protests is that rapidly developing technology is expanding the types of measurements and subsequent data available to biologists. Thus future generations of biologists will require a set of quantitative and analytic skills that will allow them to handle these types of data in order to tackle relevant questions of interest. In this spirit, we describe here strategies (or lessons learned) for undergraduate educators with regard to better preparing undergraduate biology majors for the new types of challenges that lay ahead. The topics covered here span a broad range, from classroom approaches to the administrative level (e.g., fostering inter-departmental communication, student advising) and beyond. A key theme here is the need for an attitude shift with regard to mathematics education by both students and faculty alike. Such a shift will facilitate the development and implementation of new teaching strategies with regard to improving integration of mathematics and biology pedagogy.

MOTIVATION

Recent reports have called for change in how undergraduate mathematics education is approached for students in biology (6). A compelling argument was made by Bialek and Botstein that the traditional path towards fostering quantitative biologists – having students from the physical/mathematical/engineering sciences get involved in biological problems at the graduate level (or beyond) – was no longer sufficient (5). They argued that biological sciences were getting too inherently complex to effectively learn the biology and the interconnections across various fields of study at a late stage; a more integrated approach was required early on at the undergraduate level. Another recent publication lends support (2), indicating that analytical content knowledge alone for those finishing secondary education does not necessarily correlate well to a student's scientific reasoning ability.

Clearly, there is a compelling need for future generations of biologists with strong quantitative and analytic reasoning skills (6). For example, mathematical-based models can serve to tie together the vast swaths of biological data that are increasingly coming to light. Given the complexity of biological systems, such models serve to provide a coherent and interpretable framework with which to tie together empirical observations. One example is the field of neuroscience, where models can be critical for determining future research directions (1).

The goal of this Perspectives piece is to outline a set of strategies and priorities for educators who are interested in the integration of quantitative and analytical reasoning skills into biology education by means of developing new biology–mathematics curricula (BioMath). A critical aspect for developing and implementing new strategies is facilitating an attitude shift in both students and faculty alike with regard to the perception and learning of mathematics. While the target audience is undergraduate educators, presumably much of the present discussion applies to high school and even graduate education. For details with regard to developing specific course content, several texts provide an excellent starting point at the undergraduate level (and beyond) (3, 22, 12, 9, 17, 14, 8).

The ideas described here derive from experiences with the BioMath committee at the University of Arizona. This committee, comprised of faculty from several departments as well as administrators and students, developed and implemented a new three-semester mathematics sequence offered to incoming life sciences freshmen. The courses, covering integral calculus, differential equations and (calculus-based) statistics, were designed with several goals in mind:

1. Retention of mathematical concepts and integration into subsequent science courses
2. Improvement of analytic and scientific reasoning skills
3. Shift in student's attitudes towards demanding a deeper conceptual understanding of issues encountered both inside and outside the classroom, regardless of the subject

A fourth course for upperclassmen was also developed, offered through the physiology department, that introduced students to more advanced analytic approaches to some

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fundamental topics in biology (e.g., diffusion, cell membrane electro-dynamics, the Hodgkin-Huxley model, fluid–flow in the cardiovascular system).

WHAT IS MATHEMATICS?

One pitfall that hinders efforts to bridge mathematics and biology education stems from differences in the perception as to precisely what mathematics is. Courant and Robbins (7) provide an informative starting point: ‘For scholars and layman alike it is not philosophy but active experience in mathematics itself that alone can answer the question: What is mathematics?’ While one could devote significant energy to debating a precise definition of mathematics, our purpose here is more modestly concerned with identifying some common perceptions (and misperceptions) that can strongly affect the development of undergraduate biology pedagogy.

Many texts introduce biology to students as an evolutionary science, where one is first encouraged to ask “how” and then, subsequently, “why”. [See Purves et al. (20).] Perhaps many difficulties in bridging mathematics and biology arise due to the mistaken notion that mathematics can be thought of solely as a means to establish quantitative literacy (i.e., math merely as a language). One strategy is to consider mathematics a science, one that stresses both the how and why simultaneously. Subsequently, concepts in mathematics can lead to completely new scientific insights into biological systems, not merely new ways of describing them.

In order to facilitate discussion between mathematics and biology faculty, Table I provides a broad and generalized comparison of how these fields of study are commonly perceived by both students and educators (regardless of their particular field of study). These descriptions are by no means intended to provide a definitive account as to how one should regard these fields, but merely heuristically point out perceptual differences that can exist and hinder development of BioMath efforts. Many readers will likely find that they do or do not (sometimes strongly) agree with the descriptions provided in Table I, a characteristic that hopefully will foster productive interaction across departments. It is important to note that these distinctions are becoming increasingly blurred with the rising popularity of subfields in biology education that take a more integrated approach (e.g., theoretical neuroscience, biomedical engineering, sensory physiology), as well as dedicated biomathematics programs at both the undergraduate and graduate levels.

PRIORITIES

For setting a fertile ground in terms of developing strategies and course content integrating mathematics and biology, it is important to identify (or reidentify, as the case may be) what our teaching priorities need be. Below are some notions (stemming from our experiences) that, upon discussion, can lead to fruitful directions for

course development and promote interactions amongst departments. Each is grouped according to the type of issue it addresses.

Technique - in the classroom

- ◆ Teaching mathematics should be thought of as an opportunity to develop critical thinking skills in students (i.e., ask both how and why). Mathematics courses should not be about emphasizing formulaic applications, but conceptually how to go about attacking a problem (19, 10). One example is the ubiquitous word problem. These types of problems, where students need to translate a qualitative description into an analytic one, provide an ideal opportunity to emphasize the valuable skill of transforming a problem from one domain (where it may be untenable) to another (where it is readily approachable). Furthermore, mathematics courses present a good opportunity to help students develop the ability to realize when they have made errors in their approach and need to step back and consider new strategies, an indispensable skill for biologists.
- ◆ Interdisciplinary efforts should strive not to unduly sacrifice depth for breadth. For example, one need not lose mathematical rigor by introducing a biological context to motivate a mathematical problem (e.g., consider population dynamics or disease spread to motivate systems of coupled differential equations). The converse is also true; biological content can often be enhanced by concurrently introducing relevant mathematical concepts that provide new perspectives and ways to conceptualize the system of interest (e.g., pattern formation). From our experience, the right balance between mathematical and biological focus shifts with the student’s progress through the university. Our strategy was to use biology to motivate student’s interest in freshmen courses (e.g., harmonic oscillators as auditory filters of the inner ear), but primarily place teaching emphasis on mathematical content. Conversely, the upper-division courses can place more emphasis back onto the biology as students already tend to feel comfortable with utilizing mathematics by that point (e.g., mechanisms of oxygen transport in the vascular system).
- ◆ Recitation sections should be incorporated into BioMath course curricula, thereby providing students with opportunities for conceptual inquiry in a group setting that they would not raise in the more didactic-based class session. For our courses, recitations typically either had a class discussion on a practical topic not covered in class directly (e.g., Fourier analysis and its relationship to auditory perception, random walks within the context of molecular diffusion) or gave students an open-ended problem they would work on together in small groups. Students responded favorably to these recitations as the sessions provided a chance to import and utilize recently gained classroom knowledge into

TABLE I.
Common perceptions regarding the mathematical and biological sciences

<i>Mathematics</i>	<i>Biology</i>
An old science: well-established, textbook material commonly dates back hundreds of years	A young science: textbook material is continually evolving ^a
An axiomatic science that rests upon a bedrock of established principles and seeks analytical relationships	An observational science full of qualitative and quantitative descriptions used in finding relationships
Relatively technology-independent: numerical simulations help, but general attitude favors obtaining results analytically	Relatively technology-dependent: life scientists need the tools in order to make the observations
Concise history: major breakthroughs typically attributed to a small number of key individuals	Sprawling history: many breakthroughs in biology tend to span many individuals and disciplines
Teaching faculty expertise: Broad: expected to teach any undergraduate course	Teaching faculty expertise: Specialized [typically teach solely in their area of expertise or at the introductory level (i.e., a broad overview for freshmen)]

^a In some biological fields, there is a degree of concern that material taught at the start of a four-year college will be outdated by the end of a student's undergraduate studies. How this might affect teaching strategies is unclear.

a more practical framework. Additionally, by having further chances to actively participate, students gained confidence in voicing and justifying their ideas (15).

Administrative

- ◆ As the importance of this was alluded to in the Bio2010 report (6) but never fleshed out, quantifiable assessment of changes in student performance and attitude is highly desirable. It is valuable to determine what approaches do and do not work, as well as how students view their own ability to better integrate their analytic reasoning skills into biological problems. One approach is to team up with education faculty (preferably whose expertise is in undergraduate education), who often can contribute significantly towards developing effective assessment strategies. In order to track changes in attitude as well as retention of material across subsequent courses, we developed several survey methods: questionnaires (taken at the start and end of a given course), as well as an interview focused upon a core conceptual problem (e.g., Michaelis-Menten kinetics) that allows a student to utilize mathematics skills in order to push ahead in dimensions not previously encountered in class. It is worth emphasizing that if any data acquired is to be disseminated beyond the context of the specific course development (i.e., it will be used as quantitative data in an education development forum), it is imperative to get an early start on a human studies protocol that will allow you to ask for student consent and subsequently share collected results.
- ◆ Capitalize on critical transitions in a student's life,

such as the transition from high school to college or undergraduate to graduate school. The National Science Foundation has dedicated significant resources towards this very notion via their MCTP program (Mentoring Through Critical Transition Points). These transitions potentially provide an opportunity for educators to more effectively introduce major shifts in student attitudes and perceptions. For example, it may be easier to minimize biases against mathematics (that might arise from the ubiquitous drill problems students have likely encountered) when a student is new to the university environment and they are getting exposed to new ways of thinking. A frequent comment we received from freshmen was that our courses were nothing like any other mathematics course they had taken in high school in that the emphasis was now upon conceptual understanding and critical thinking, rather than repetitive problem sets.

Strategic – beyond the classroom

- ◆ Motivated by the Bio2010 report (6), it is important to consider the notions of a “quantitative biologist” (one at the interface of biology and mathematics/computer science) versus a “research biologist” (one who can converse productively with mathematicians and computer/physical scientists, but need not be an expert in those fields, per se) (21). A student's attitude may gravitate more towards one or the other, based upon abilities and interests. Such inclinations may be apparent early on for some undergraduates, but may develop later on for others after exposure to new ideas and approaches

from coursework and research experience. Identifying a student's preferred leanings as they evolve and directing him or her towards an appropriate set of courses is highly desirable, thus careful advising is of great value. If one merely looks at a list of departmental requirements and chooses a schedule based upon that alone, the student may miss out on the opportunity to take courses that might better fit his/her interests and strengths. At the University of Arizona, multivariable calculus is a commonly recommended (or even required) course for students in majors such as biochemistry and ecology and evolutionary biology. But these students are likely better off taking a course on ordinary differential equations (ODEs) which, on the most basic level, describe how something changes with respect to something(s) else. From our experience, students tend to change majors quite a bit, particularly within the first two years of being at the university (which is usually when BioMath courses should be introduced!). Ideally, BioMath courses should be designed so that a student would not be penalized if, for example, he/she changed majors from life science to mathematics or from biochemistry to physiology.

- ◆ Unfortunately, it is generally true that if you don't use it, you lose it. Thus a potentially useful consideration is the development of mathematics refresher sessions for biology faculty. Such events could be developed locally or within the framework of preexisting organizations. [See the Faculty Programs Institutes (13) and the Mathematics Bioscience Institute (18).] These refreshers would provide faculty with a means to revisit topics they likely have not seen in some time, as well as potentially learn new methods that may be of use in their current research. Furthermore, such sessions would give biology faculty a greater degree of confidence when mentoring students who want to work with them on more analytical-based approaches to their area of study. In addition, participation can provide instructors with ideas about how to introduce mathematical content into their life science courses. Faculty not needing a refresher could make ideal candidates to organize such sessions and motivate people to come. Running these sessions would also provide further opportunity for interaction between biologists and mathematicians. The value of the converse should also be noted: mathematics faculty would surely benefit from a review of key concepts in biology as well as biological research methods!
- ◆ An undergraduate research experience is critical towards facilitating the integration of mathematics and biology. Such experiences provide a student with a scientific problem into which they are deeply and personally invested (4). These problems commonly require various degrees of utilizing mathematical knowledge, such as statistics and modeling. The research environment, where questions are typically open-ended and do not have clear well-defined answers, provides students with a chance to develop critical thinking skills and, in turn, stimulates them

to seek courses that will likely be of value to address these questions. From our experiences, students who either are currently involved in research or plan to do so in the summer are significantly more motivated with our courses than students who are not involved with a research experience (4). For example, most freshmen tend not to see the value in taking a concept-oriented statistics course. However, after a research experience in a lab where they need to draw conclusions from a set of observations (and subsequently convince others of said conclusions), the power of the tools that a statistics course offers becomes apparent. Students readily commented that material learned in the classroom is greatly reinforced when they apply those ideas in the laboratory.

Pitfalls

- ◆ When developing new courses, careful consideration needs to be given with regard to how such courses will affect subsequent prerequisites for future classes a student will take. For example, the amount of focus upon ODEs for freshmen should depend upon how much students may encounter ODEs in upper-division courses. In bioinformatics, for example, one might argue that discrete mathematics (which encompasses such topics as logic, set theory and combinatorics) may be of greater value than ODEs. However, it may well be that the specific content (e.g., ODEs versus discrete math) is not critical. But what is essential is providing students with a foundation that gives them a core set of problem-solving skills and an attitude that allows them to readily adapt in the future when faced with problems with which they do not know how to deal. These two aspects are critical ingredients for scientists who can excel in an independent environment. Put another way, BioMath courses should strive to impart to students the skill of being able to self-identify mathematical deficiencies relevant to their research and shore them up, either via a class or self-study.
- ◆ Avoid drill problems. Many students entering the university typically associate mathematics with long sets of repetitive problems, most likely stemming from preparation for standardized tests such as the Advanced Placement exam. Ideally, there needs to be a balance between abstract conceptualization and generalized examples. Problems should be structured in such a way as to encourage critical thinking, not formulaic routine. Furthermore, in-class examples should encourage active participation, thereby allowing each individual student to contribute to the discourse and stay engaged. [See Mazur (16).] We have observed that freshmen likely found the transition away from drill problems to be a bit of a shock, making it essential to introduce this shift as early as possible and encourage students that (hard-fought) dead ends can be just as rewarding as correct solutions when faced with a challenging problem (10).
- ◆ Given the broadness of BioMath efforts, it is important

to be aware of cultural differences that may exist across departments. One ideally does not want to alienate other departments that can readily work towards the common cause. Fostering interaction amongst departments was one of the more challenging hurdles we faced. Different departments have different teaching priorities, resources, and philosophies. Thus it is not always easy to find common ground. However, we feel the interactions among faculty from different departments are critical, given the interdisciplinary spirit of BioMath development.

- ◆ Another factor in BioMath development is the subjective nature of course-effectiveness that is dependent upon individual instructors. Ideally, curriculum development can minimize this effect by providing instructors with some degree of flexibility to shape topics as they see fit. This flexibility helps ensure that instructors stay engaged with the course material, since they have freedom to focus on topics personally of interest. For example, faculty with a biochemistry background might use the law of mass action and Michaelis-Menten kinetics to motivate systems of coupled ODEs and dynamical systems theory. One strategy stemming from our experience is to involve more than one faculty member in course development and teaching – preferably one person from mathematics and another from the life sciences. Even if one person's role is relatively limited, regular meetings to discuss ideas for course material can have a significant impact.

SUMMARY

As the field of biology rapidly evolves in the face of new technology and data acquisition methods, quantitative and analytical skills (rooted in a solid foundation of mathematical knowledge) will be of ever-increasing importance. Many challenges face educators who are working towards addressing the need of better developing these skills in future generations of biology students (11). These difficulties arise both inside and outside of the classroom and require significant investment from multiple departments across the campus. In the broadest sense, many of the points raised here are centered around instituting a shift in attitude towards BioMath in both students and faculty alike. From our experiences with regard to the faculty point of view, identifying one's own perceptions with respect to mathematics and how they compare to those of their colleagues can go a long way towards setting a common ground for course development and implementation. Furthermore, overcoming student biases against mathematics can provide fertile ground for developing critical thinking skills and allowing students to demand a deeper understanding of course material (i.e., ask both how and why) that instructors can then capitalize upon in the classroom.

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