C. Bergevin
8.28.09

Outline for Each Semester
[items in italics are optional depending upon timing, but represent desirable topics to cover]
---> # of classes listed between exams excludes Tuesday afternoon sessions (just MWF classes)
++

250A (Fall 2009)

[class starts 8/24; **10 classes to exam I**]

A1. Review material they should know coming in (dealing w/ analytic expressions, differentiation, definite integrals)
- definition of a function, limit, continuity, differentiability
- types of functions: linear, polynomial, exponential, logarithmic, rational, trigonometric
- using calculus to determine the behavior of functions (e.g., critical points, concavity, limits)
- basics of problem-solving & critical thinking
- methods of differentiation

A2. Introduce and motivate ODEs and what they can be used for, types of ODEs
- incorporate via the notion of 'word problems'
- expose to a diverse range of ODE applications to motivate
- different forms of 1st order ODEs:
    - dy/dx = f(x,y) : general form
    - dy/dx = f(y) : autonomous (general type we will approach early on)
    - dy/dx = f(x)g(y) : solvable via separation of variables
- 2nd and higher order eqns. (e.g., Newton's second law)
- systems of coupled ODEs
- expand upon linear vs nonlinear ODEs
- ----> our focus will be upon linear eqns. where we can do some degree of analysis
    [nonlinear cases will come later, where for the most part we try to linearize!]

A3. Preliminary means to solve ODEs
- solving ODES <---> integration
- separation of variables
- ---> motivates methods of integration

A4. Numerical methods to solve definite integrals (i.e., Riemann sums) **(LAB)**
- first exposure to using the computer as a programmable tool
- methods: LHR, RHR, Midpoint rule, trapezoid rule, Simpson's rule

EXAM1 (9/16/09)

[**11 classes to exam II**]

B1. Methods of Integration
- Integration by parts
- Integration via substitution
- Using the table of integrals

B2. Definite integrals within the context of solving ODEs (see 2008 exam II)

B3. Qualitative aspects of dealing with differential eqns.
- classifying systems (e.g. linear vs nonlinear, autonomous, etc...)
- slope fields **(LAB)**
- finding equilibrium points and identifying stability
- initial/boundary conditions (leads in exist. & uniqueness)
- symmetry/behavior of solutions

EXAM2 (10/14/09)

[**16 classes to exam III**]

C1. Existence & Uniqueness of solutions to ODEs

C2. More advanced methods of integration
- trig. substitutions
- partial fractions (motivate via solving the logistic equation)

C3. Euler's method to numerically solve ODEs **(LAB)**

C4. Phase-line diagrams for ODEs, notion of bifurcations (affecting stability)

C5. Linear regression
- *actually using calculus to find slope/intercept given a set of points*
- *when fitting to a straight-line might not be such a good idea*
- *allude to nonlinear regression as well as loess (i.e., non-model specific curve fitting)*

EXAM3 (11/25/09)

[5 classes to final, including the last day]

D1. Introduce methods to solve linear ODEs (Lomen & Lovelock Ch.5)

D2. Examples of ODEs
- exponential growth/decay
- logistic growth

FINAL

Semester 1 Labs

A. Write a code to implement numeric approx. of integrals via Riemann sums
B. Use dfield in Matlab to manipulate slope fields for certain first-order ODEs
C. Use Euler's method to solve ODEs

++

250B (Spring 2009)

A1. Taylor polynomials (as a means of approximating functions)
A2. Improper integrals
A3. Integral applications
- arc length
- volumes
- solids of revolution

EXAM1

B1. Sequences (e.g., convergence)
B2. Infinite Series
- convergence
- interval of convergence
- geometric series

B3. Power series
B4. Taylor series expansions
- estimating associated error

B5. Notion of linear independence
B6. Fourier series
- *applications to hearing*
- *as another means to solve ODEs*

EXAM2

C1. Complex #s
- complex arithmetic
- magnitude and phase <--> cartesian coordinates
- connection back to hyperbolic trig. functions

C2. 2nd order ODEs (homogeneous and nonhomogeneous) & Systems of ODEs
- notion of the phase plane
- general examples
- harmonic oscillator [(un)forced, (un)damped, active/passive]
- *connection back to Fourier series*
- Variation of Parameters
- Method of Undetermined Coefficients

C3. Converting higher order eqns. into systems of 1st order eqns.
C4. Power series solutions to ODEs
C5. Variation of parameters

EXAM3

D1. Coupled ODEs
- *application to Michaelis-Menten kinetics* ***(LAB)***

D2. Nonlinear systems
- notion of orbits and relationship to critical points & stability
- Eigenvalues from linearized system to classify stability
- Bifurcation parameters

FINAL

<u>Semester II Labs</u>

A2?. Exercise on linear regression?
A2?. erf lab
B2. Jim's dynamical systems lab re. Michealis-Menten kinetics

++
++

<u>Key Concepts Students Should Come Away With from 250A&B (2009-2010)</u>

- Comfort dealing with analytic representations of quantities/functions
    - o linear, polynomial functions
    - o logarithms
    - o exponential functions
    - o trigonometric functions (including hyperbolic versions)
    - o plotting functions (critical points, inflection points, limits, etc....)
- Methods of differentiation
- Methods of integration
    - o definite integrals
    - o improper integrals
    - o using trig. substitutions
    - o integration by parts
    - o numerical approaches via Riemann sums **(LAB)**
- Complex #s
    - o complex arithmetic
    - o magnitude and phase <--> cartesian coordinates
    - o connection back to hyperbolic trig. functions
- 'Word problems'
- Using ODEs as a means to describe phenomena
- Qualitative approaches to ODEs

- classifying systems (e.g. linear vs nonlinear, autonomous, etc...)
    - slope fields
    - finding equilibrium points and identifying stability
- Methods for solving ODEs
    - relation back to integrals (for separable eqns.)
    - implementing numerical means to solve via Euler's method **(LAB)**
    - series approxs. (see below)
    - eigenvalues (including complex cases)
- Sequences
- Series
    - infinite series
    - convergence, intervals of convergence
    - geometric series
    - power series
- Taylor series approximations
    - notion of linear independence
    - application to ODEs
- *Fourier series*
- Systems of ODEs/2nd Order ODES & Methods to Solve
    - Variation of Parameters
    - Method of Undetermined Coefficients
- Introduction to nonlinear systems
    - introduction to the phase plane
    - notion of orbits and relationship to critical points & stability
    - *application to Michaelis-Menten kinetics **(LAB)***
- *Linear regression*