What to Say About the Tacoma Narrows Bridge to Your Introductory Physics Class

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Having just taught my introductory physics class about the collapse of the Tacoma Narrows Bridge, I realized that this topic is fascinating both to the students and to me. I always show the video and it never fails to elicit numerous questions from the class. However, its treatment in most introductory physics textbooks is either at best inadequate or at worst misleading. By chance, I also recently heard a talk from the project manager of a proposed new bridge across the Mississippi River at St. Louis. The issues that led to the failure of the Tacoma Narrows Bridge played a major role in the design of this new bridge. These two events have led me to think about the physics of bridge oscillations and to write this paper, which is in large part an abridged version of the 1991 *American Journal of Physics* article by Billah and Scanlan,[1] but it is clear to me that the content of that article has not permeated the physics community.

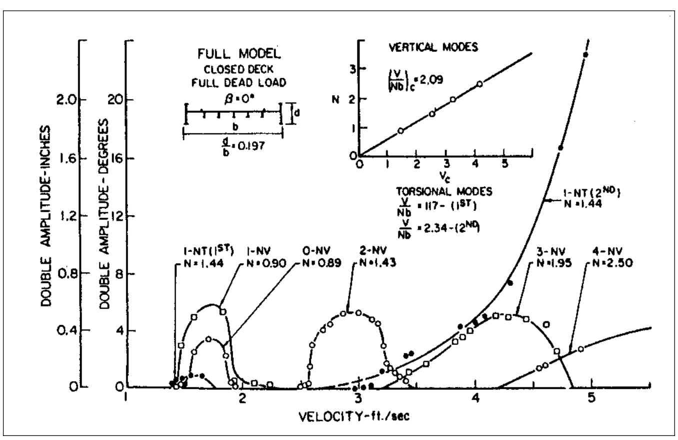

Fig. 1. Wind-tunnel results from a 1/50th-scale model Tacoma Narrows Bridge.[3] The wind velocity in the wind tunnel of 5 ft/s (1.5 m/s) is equivalent to a wind velocity of 35 mph (16 m/s) for a full-size bridge.

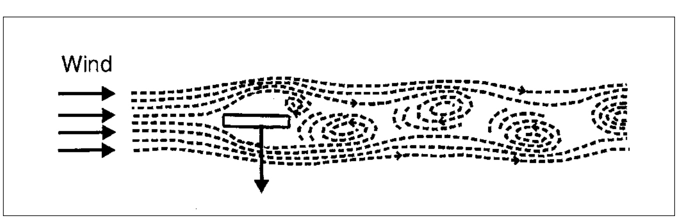

Fig. 2. A simple schematic of the vortex array formed when a constant wind blows around a bridge deck.[2] The downward arrow is the direction of the force exerted on the deck by the creation of the vortex above the bridge deck.

Tacoma Narrows Bridge Vertical Oscillations

If you watch the video of the collapse of the Tacoma Narrows Bridge,[2] you will observe two very different types of bridge oscillation. Up until an hour before the collapse of the bridge, the only observed oscillation was a vertical motion of the bridge deck. This vertical oscillation is an example of simple forced harmonic oscillation — the sinusoidal motion of the deck due to an external sinusoidal force. The solution of Newton's second law gives the steady-state motion of the bridge bed as a function of time and can be found in Appendix A.

This steady-state solution displays two defining characteristics of forced harmonic motion. One is that the bridge deck oscillates at ω_{ex}, the angular frequency of the external sinusoidal force, not at ω_0, the natural angular frequency of the bridge bed. Second,

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the amplitude of the oscillation has a maximum when $\omega_{ex} = \omega_0$ (in other words, a resonance behavior as a function of wind velocity since ω_{ex} is proportional to the wind velocity — see below).

The vertical oscillations of a model Tacoma Narrows Bridge in a wind tunnel display both characteristics of forced harmonic motion. First, the frequency of the vertical oscillations varies linearly with the wind velocity. Second, Fig. 1 shows the amplitude of various oscillations as a function of wind velocity; notice that curves 1-NV and 2-NV show the expected resonance behavior.[3]

The oscillating external force involved in these vertical oscillations is due to vortex shedding by the constant velocity wind separated by the deck of the bridge. These vortex arrays, commonly called a Karman vortex street and sketched in Fig. 2, are two rows of vortices with opposite directions of circulation and with a frequency that is approximately proportional to the wind velocity. At the time a vortex breaks loose from the top (bottom) of the bridge deck, a downward (upward) force is exerted on the bridge deck, as sketched in Fig. 2.

A number of everyday occurrences are explained by this phenomenon of vortex shedding. For example, automobile radio antennae will oscillate perpendicular to the motion of a car at certain resonant velocities. In heavy winds, stoplights strung across streets will oscillate perpendicular to the direction of the wind. A simple classroom demonstration of vortex shedding is the motion of a sheet of paper that is dropped to the floor; each flutter of the paper is one vortex being shed.

Tacoma Narrow Bridge Torsional Oscillations

In the last 45 minutes of the life of the Tacoma Narrows Bridge, a new twisting, torsional motion is observed, which has a very large amplitude and is the cause of the bridge's collapse. This oscillation is an aerodynamically induced self-excitation (or aerodynamic flutter). The solution to Newton's second law for a self-excited system also gives a steady-state sinusoidal solution, which can be found in Appendix B.

The two characteristics of this steady-state solution are motion at the natural frequency ω_0 (not at ω_{ex}, since there is no external sinusoidal torque), and no resonance behavior in the amplitude as a function of the wind velocity. The torsional motion of the model Tacoma Narrows Bridge in a wind tunnel has both these characteristics. First, the frequency of the torsional motion does not change with wind velocity. Second, returning to Fig. 1, the curve 1-NT is due to the torsional motion and shows no resonance behavior.

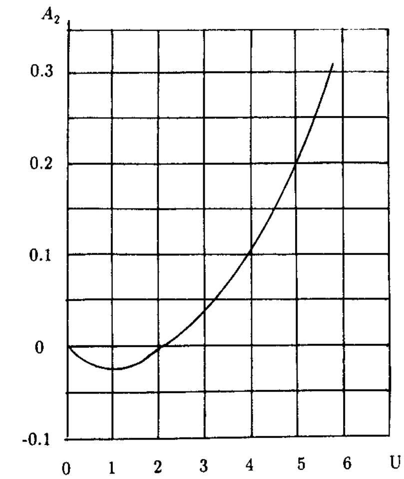

Fig. 3. The values of A_2 as a function of wind velocity, *U*, from wind tunnel measurements.[4] A *U* of 5 is approximately equivalent to a wind velocity of 27 mph (12 m/s) for a full-size bridge.

A further insight into the self-excited motion comes from the transient solution to Newton's second law, also found in Appendix B. The amplitude of the transient solution is exponential in time, where the exponent is a function of the aerodynamic flutter coefficient, A_2, and the torsional damping coefficient, b. This exponent can be either positive or negative, depending on the magnitude and sign of A_2 and b.

Scanlon and Tomko fitted A_2 to the transient motion of a model Tacoma Narrows Bridge in a wind tunnel.[4] The experimental values of A_2 as a function of wind velocity U are shown in Fig. 3. At wind velocities less than 10 mph (4.5 m/s), A_2 is negative, the exponential coefficient is negative, and the torsional motion decays in time. However, above 27 mph (12 m/s), A_2 is positive and greater than b, the exponent becomes positive, and the torsional motion grows exponentially in time. From the video, the Tacoma Narrows Bridge was in a steady-state mode most of the 45

Fig. 4. Computer-generated photograph of the new Mississippi River Bridge proposed for St. Louis, courtesy of the Illinois Department of Transportation.

minutes of torsional motion. My guess is that as the wind increased in speed, the amplitude initially grew exponentially, but as the amplitude increased, so did b until a new steady-state amplitude was reached.

The really difficult question is the physical mechanism behind the behavior of A_2 shown in Fig. 3. Billah and Scanlon suggest that the torsional motion of the bridge generated a vortex wake that amplified the torsional motion.[1] They call this an "aeronautically induced condition of self-excitation."

However, there are other proposed explanations.[2] Lazer and McKenna proposed a nonlinear model where first the wind drove the bridge cables into a high-frequency oscillation; then a nonlinear mechanism channeled that energy into the low-frequency torsional mode.[5]

The New Mississippi River Bridge

By coincidence, as I was lecturing my class about the Tacoma Narrows Bridge, I heard a talk by Keith Hinkebein from HNTB Corp. about the design of the proposed new Mississippi River Bridge in St. Louis. Hinkebein is the project manager for this endeavor, which is sponsored by the Illinois Department of Transportation and the Missouri Department of Transportation. A computer-generated photograph of the proposed bridge, courtesy of the Illinois Department of Transportation, is shown in Fig. 4. Naturally, I asked him how this bridge had been designed to withstand high winds and tornadoes.

One can first ask what does the simple analysis in this paper suggest about how to improve bridge stability. In the case of forced harmonic oscillations, if the bridge is made much more rigid, the natural frequencies of the bridge can be increased so that they are greater than the Karman wave frequency of very high winds. Consequently, most modern bridges are much more rigid than the Tacoma Narrows Bridge.

In the case of self-excited amplification, the goal is to increase b, so it is greater than any possible A_2. In the case of the new Tacoma Narrows Bridge, hydraulic dampers were installed at the towers and piers to increase b.[6] High torsional stiffness is also essential for bridge stability, since it shifts the self-excited oscillations to much higher wind velocities.[4]

In the case of the new Mississippi River Bridge, the engineers relied on both theory and wind-tunnel measurements to ensure the stability of this bridge. The most serious wind-induced oscillation of this new bridge was an aerodynamic self-excitation mode, just as in the case of the Tacoma Narrows Bridge. The major new design feature used to suppress this self-excitation mode is a wide (12-m) slot between the two decks of the bridge, which tends to equalize the pressure above and below the bridge deck.

What an Instructor Should Say

For all high school and many college physics courses, the treatment in the appendices using differential equations is far too advanced. However, the instructor can give a qualitative explanation as follows: There were two types of oscillation of the Tacoma Narrows Bridge. The vertical oscillation is an example of forced harmonic oscillations, where the vortex train created by the wind separated by the bridge deck is the sinusoidal external force. The major characteristics of forced harmonic oscillation are that the frequency of the vertical oscillation of the bridge deck is at the vortex shedding frequency, and the amplitude of the oscillation has a maximum when the vortex shedding frequency equals the natural vertical frequency of the bridge deck. In contrast, the torsional oscillation is an aerodynamically induced self-excitation phenomenon, where the amplification mechanism is still being debated by engineers. The major characteristics of self-excitation oscillations are that the oscillation frequency is the natural torsional frequency of the bridge deck, and there is no resonance phenomenon in the amplitude as a function of wind velocity.

Conclusions

Billah and Scanlon[1] made a very astute observation in 1991 that is still valid today. They point out that the mystery of the collapse of the Tacoma Narrows Bridge is not a mystery; Farquharson in 1950 gave the correct explanation from wind tunnel studies.[3] The real mystery is why the physics community has not taught the correct explanation for all the years since 1950, and may I add, since the publication of the *AJP* article by Billah and Scanlan in 1991. My hope is that this paper will help end this mystery. My other hope is that the real mystery, the lack of understanding of the physical mechanism for self-excited oscillations, may in a small way inspire future physicists and engineers. My experience is that excitement and interest in physics and engineering is generated not only by what is understood but also by what is not.

Appendix A. Solution to Forced Harmonic Motion Equation

The differential equation for the sum of the forces acting on a mass attached to a spring driven by a sinusoidal external force is

$$m\ddot{x} + b\dot{x} + kx = F_{ex}\exp(i\omega_{ex}t)\,, \tag{1}$$

where m is the mass; b, the spring damping constant; k, the spring constant; F_{ex}, the amplitude of the external force; and ω_{ex}, the angular frequency of the external force. Because of the use of complex functions, the external force is the real part of $F_{ex}\exp(i\omega_{ex}t) = F_{ex}\cos(\omega_{ex}t)$ and the position of the spring is the real part of x. Dividing by m gives

$$\ddot{x} + (b/m)\dot{x} + \omega_0^2 x = (F_{ex}/m)\exp(i\omega_{ex}t), \tag{2}$$

where $\omega_0 = (k/m)^{1/2}$ and is the angular frequency of the undamped oscillator and is called the natural angular frequency. The steady-state solution of Eq. (2) is

$$x = A\exp(i\omega_{ex}t), \tag{3}$$

where

$$A = (F_{ex}/m)/[\omega_0^2 - \omega_{ex}^2 + i(b/m)\omega_{ex}]\,. \tag{4}$$

To get the position of the spring, the real part of x can be calculated with a little bit of algebra.

$$\text{Real}\,(x) = (F_{ex}/m)\,[(\omega_0^2 - \omega_{ex}^2)^2 + (b/m)^2\omega_{ex}^2]^{-1/2}\cos(\omega_{ex}t + \phi), \tag{5}$$

where ϕ is the phase angle between the external force and the motion.

Appendix B. Solution to Aerodynamic Self-Excitation Equation

The differential equation for the sum of the torques acting on the bridge that is aerodynamically self-excited is

$$I\ddot{\theta} + b\dot{\theta} + (\omega_0^2 I)\theta = A_2\dot{\theta} + iA_3\theta\,, \tag{6}$$

where θ is the angle of twist of the bridge; I, the bridge's moment of inertia; b, the torsional damping coefficient; ω_0, the undamped angular frequency of the torsional mode; and A_2 and A_3, the flutter coefficients, which have a strong dependence on the wind velocity.[1,4] Equation (6) is very similar to Eq. (1), except there is no external sinusoidal torque; instead, the driving torques are $A_2\theta + iA_3\dot{\theta}$, which only depend on the motion of the bridge and the velocity of the wind, and which is why the motion is called aerodynamical self-excitation.

The steady-state solution to Eq. (6) is

$$\theta = A\exp(i\omega_0 t), \tag{7}$$

where

$$(b - A_2)\omega_0 - A_3 = 0. \tag{8}$$

A further insight into Eq. (6) comes from its transient behavior. The transient solution is

$$x = A\exp(-\alpha t)\exp(i\omega t). \tag{9}$$

Plugging this solution into Eq. (6) gives

$$2I\omega\alpha = (b - A_2)\omega - A_3 \tag{10}$$

from the imaginary part and

$$\omega_2 - \omega_0{}^2 - \alpha^2 + (1/I)(b - A_2)\alpha = 0 \tag{11}$$

from the real part. Notice that if α is positive, any small torsional oscillation will exponentially decay. However, if α is negative, then any small torsional oscillation will grow exponentially. At all of these wind velocities, A_3 can be neglected.[4]

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