CHAPTER 11

Basic MATLAB Programming

MATLAB is a matrix-based language. Since operations may be performed on each entry of a matrix, "for" loops can often be bypassed by using this option. As a consequence, MATLAB programs are often much shorter and easier to read than programs written for instance in C or Fortran. Below, we mention basic MATLAB commands, which will allow a novice to start using this software. The reader is encouraged to use the Help Graphic User Interface (GUI) for further information.

1. Defining a row matrix and performing operations on it

Assume that you want to evaluate the function $f(x) = x^3 - 6x^2 + 3$ at different values of x. This can be accomplished with two lines of MATLAB code.

```
% Define the values of x
x = 0:0.01:1;
% Evaluate f
f = x .^ 3 - 6 * x .^ 2 + 3;
```

In this example, x varies between 0 and 1 in steps of 0.01. Comments are preceded by a % sign. The symbols ˆ and * stand for the power and multiplication operators respectively. The dot in front of ˆn indicates that each entry of the row matrix x is raised to the power n. In the absence of this dot, MATLAB would try to take the nth power of x, and an error message would be produced since x is not a square matrix. A semicolon at the end of a command line indicates that the output should not be printed on the screen.

Exercises:

(1) Type `size(x)` to find out what the size of x is.
(2) Evaluate the cosine (type `y = cos(x);`) and the sine (type `z = sin(x);`) of x.
(3) Define a square matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},$$

by typing `A = [ 1 2 ; 3 4 ]`. Then compute the square of A (type `A ^ 2`), and compare the result to that obtained by typing `A .^ 2`.

2. Plotting the graph of a function of one variable

The command `plot(x,f)` plots f as a function of x. The figure can be edited by hand to add labels, change the thickness of the line of the plot, add markers, change the axes etc. All of these attributes can also be specified as part of the `plot` command (type `help plot` or search for "plot" in the Help GUI for more information).

Exercises:

(1) Plot
- $\sin(x)$ (type `plot(x,sin(x))`) and
- $\cos(x)$ (type `plot(x,cos(x))`).

To plot both curves on the same figure, type
```
plot(x,cos(x))
hold
plot(x,sin(x)),
```
or type
```
plot(x,sin(x),x,cos(x)).
```
(2) Plot the graph of $\exp(x)$ for $x \in [-5, 10]$.

3. Basic vector and matrix operations

An n-dimensional vector u in MATLAB is a $1 \times n$ matrix, whose entries can be accessed by typing `u(j)` where j is between 1 and n. For instance, if you want to define a vector v whose entries are $u(10)$, $u(11)$, ..., $u(20)$, type `v = u(10:20);` (recall that a semicolon at the end of a MATLAB command indicates that the output of that command should not be displayed on the command window).

The transpose u^T (in MATLAB, type `u'`) of u is a column vector. To calculate the scalar product of u with u^T, type `u * u'`. If you type `u' * u`, you obtain an $n \times n$ matrix since you are multiplying an $n \times 1$ matrix by a $1 \times n$ matrix.

We have already mentioned how to raise each entry of u to a given power. You can in fact apply any function to each entry of u. For instance, `exp(u)` will return a vector whose entries are obtained by applying the exponential function to each entry of u. If A is a square matrix, you may want to calculate its exponential

$$e^A = \sum_{i=0}^{\infty} \frac{1}{i!} A^i.$$

For this, MATLAB has a special function called `expm`. Similarly, `sqrtm` will calculate a square root of a non-singular square matrix.

Exercises:

(1) Show that if a matrix M can be written as $M = P^{-1}DP$, where D is diagonal and P is invertible, then

$$\exp(M) = P^{-1}\exp(D)P.$$

(a) Define the matrices

$$P = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}, \qquad P^{-1} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix},$$

and $M = P^{-1}DP$. Enter these matrices into MATLAB.

(b) Use MATLAB to calculate $\exp(M)$ and compare the result to $P^{-1}\exp(D)P$.

(2) The MATLAB function `eig` returns the eigenvalues of a square matrix M, `det` returns its determinant and `trace` its trace.

(a) Find the eigenvalues of D and M defined above. Does MATLAB give you the right anser?

(b) Find the product and the sum of the eigenvalues of

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

4. Plotting the graph of a function of two variables

Assume that we want to use MATLAB to plot the graph of $f(x, y) = x^2 - 3y^2$, for $x \in [-3, 3]$ and $y \in [-5, 5]$. We first need to define a numerical grid where the function f will be evaluated. To this end, define the matrices x and y,

```
x = -3:0.01:3;
y = -5:0.01:5;
```

and define the numerical grid as

```
[X,Y] = meshgrid(x,y);
```

Then, evaluate f at the points on the grid and put the result in a matrix Z

```
Z = X .^ 2 - 3 * Y .^ 2;
```

Finally, plot the graph of f with the following command

```
surf(X,Y,Z), shading interp
```

The surface can be rotated by typing `rotate3D`, or by clicking on the rotation icon on the figure.

Exercises:

(1) Plot the graph of $f(x) = \exp(-2x^2 - 3y^2)$. Choose appropriate intervals for x and y.
(2) Plot the graph of $f(x) = \cos(x)\sin(y)$. Choose appropriate intervals for x and y.
(3) Change the color map of one of the plots above by using the commands `colormap bone` or `colormap jet` or `colormap cool`. Search `colormap` in the Help GUI to find more examples.