

Assignment 3

Probability

Math 363

October, 2009

1. A lake contains 26 lake trout. Twelve are captured, tagged, and released. A certain time later, 5 of the 26 are captured. What is the probability that exactly one of them is tagged? State clearly what assumptions you are using.
2. 5% of the men and 0.25% of the women are color blind.
 - (a) Assuming that the population is made up 1/2 men and 1/2 women, find the fraction of the population that is color blind.
 - (b) If a person chosen at random is color blind, find the probability that the person is male.
 - (c) If two randomly chosen individuals are color blind, find the probability that both are male.
3. Let X be a discrete random variable with probability mass function

x	1	2	3	4	5
$f_X(x)$	c	$2c$	$3c$	$2c$	c

- (a) Find c .
 - (b) Find $P\{X \leq 3\}$
 - (c) Draw the cumulative distribution for X .
 - (d) Find EX , $EX(X - 1)$ and $\text{Var}(X)$.
4. For the random variable X .
 - (a) Show that
$$F_X(x) = \frac{1}{1 + \exp(-x)}, \quad x \in \mathbb{R}.$$
is a valid cumulative probability distribution function.
 - (b) Find the median of X .
 - (c) Find the probability density for this distribution function.
 - (d) Find the probabilities $P\{-1 < X \leq 0\}$, $P\{0 < X \leq 1\}$, and $P\{-1 < X \leq 1\}$.

5. The **Pareto random variable** with parameters $\alpha > 0$ and $\beta > 0$ has probability density function

$$f_X(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad \alpha < x < \infty.$$

- (a) Verify that f_X is a density function.
 - (b) Find $P\{\alpha < X \leq 2\alpha\}$.
 - (c) Find the mean and variance of X . What restriction do you have on β in computing the variance?
6. In this problem, we shall use R to calculate and simulate with random variables.
- (a) For X a negative binomial with $n = 3$ and $p = 3/4$, find $P\{X = x\}$ for $x = 0, 1, \dots, 10$.
 - (b) For X a gamma random variable with $\alpha = 4$ and $\beta = 2$, find $P\{X \leq x\}$ for $x = 0, 1, 2, 3$. Indicate these values on a plot of the cumulative distribution function.
 - (c) For Z a standard normal, find values for z so that $P\{Z \leq z\} = 0.05, 0.25, 0.50, 0.75, 0.95$. Indicate these values on a plot of the cumulative distribution function.
 - (d) Simulate 1000 independent beta random variables with $\alpha = 2$ and $\beta = 4$. Find the mean and variance of this sample and compare it to the actual values.
 - (e) Compare the probability mass functions of a $Bin(200, 1/100)$, and $Bin(2000, 1/1000)$ and $Pois(2)$ random variable by displaying them in a `data.frame`.