

# The Expected Value\*

## Worksheet 10

1. Let  $X$  and  $Y$  be discrete random variables with mass functions

$x$	1	2	3	4	5	6	$y$	1	2	3	4	5	6
$f_X(x)$	0	0.1	0.2	0.3	0.3	0.1	$f_Y(y)$	0.2	0.2	0.2	0.2	0.2	0

- (a) Find  $P\{X \leq 3\}$ .
  - (b) Find  $P\{Y > 2\}$ .
  - (c) Find  $EX$  and  $EY$ .
  - (d) On a common graph, draw the survival function for  $X$  and  $Y$ .
  - (e) Find the area between the survival functions and check that it is equal to  $E[X - Y]$ .
2. The **Pareto random variable** with parameters  $\alpha > 0$  and  $\beta > 0$  has probability density function

$$f_X(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad \alpha < x < \infty.$$

- (a) Verify that  $f_X$  is a density function.
- (b) Find  $P\{X > 2\alpha\}$ .
- (c) Find the mean and variance of  $X$ . What restriction do you have on  $\beta$  in computing the mean and variance (a different restriction for each)?
- (d) Use the probability transform to simulate 1000 Pareto random variables with  $\alpha = 1$  and  $\beta = 4$  and find their sample mean and variance. Compare this to the values in part (c).

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