Introduction

Homework 1

Problems

1. The notation $\sigma(\mathcal{C})$ is used to denote the smallest σ-algebra generated by the collections of sets $\mathcal{C}$. Let A and B be subsets of a sample space Ω.
   (a) Find the number of distinct sets in $\sigma(\mathcal{C})$ where $\mathcal{C} = \{\mathcal{A}, \mathcal{B}\}$
   (b) List those sets.
2. Let
$$\left\{ {n \atop k} \right\}$$
   denote the Stirling numbers of the second kind. $\left\{ {n \atop k} \right\}_{1 \le k, n \le 4}$ is the matrix representation for the linear transformation
$$((x)_1, (x)_2, (x)_3, (x)_4) \to (x, x^2, x^3, x^4).$$
   Find the inverse of this matrix, These are known as the Stirling numbers of the first kind.
3. Casella & Berger, Exercise 1.7, page 38.
4. Casella & Berger, Exercise 1.18, page 39.
5. Casella & Berger, Exercise 1.20, page 39.

Challenging Problems

1. Assume that Ω is uncountable and let $\mathcal{F}$ be the collection of all subsets so that either A or A^c are countable.
   (a) Explain how $\mathcal{F}$ is a σ-algebra.
   (b) Define
$$P(B) = \begin{cases} 0 & \text{if B is countable} \\ 1 & \text{if B is not countable} \end{cases}$$
   Show that P is a probability.
2. Casella & Berger, Exercise 1.12, page 39.