Conditional Probability and Bayes Theorem

Homework 2

Problems

1. Draw 20 dates out of a hat that has each of the 365 days of the year. Find the probability that
   (a) none of the dates are in September.
   (b) two of the dates are in September.
   (c) x of the dates are in September. What values for x are possible?
2. An urn has 4 red, 5 white, and 7 blue marbles. Assume equally likely outcomes.
   (a) Find the probability of drawing, without replacement, red, red, white, white, blue, blue in that order.
   (b) Find the probability of drawing, without replacement, two red, two white, and two blue.
   (c) Repeat (a) and (b) sampling with replacement.
3. If A and B are independent, show that A^c and B^c are independent.
4. With the monsoon season, we can have more cases of dengue fever, a mosquito-borne tropical disease caused by the dengue virus. Antibody tests are recommended during a dengue outbreak. However, the presence of other viruses in the human body can have cross-reactive results yielding a high false positive rate. Assume a false positive rate of 10% and a false negative rate of 1%.
   (a) Given that a person has dengue, what is the probability of a positive test.
   (b) If one percent of a population has dengue, what fraction of the population will test positive.
   (c) If the individual tests positive, what is the probability that this individual has dengue?
   (d) The public health department suggests aggressive screening so that half of those tested have dengue. In this case, what is the probability that an individual testing positive actually has dengue?
   (e) So that the public health department can decide on the aggressiveness of the screening, provide a plot of p, the prior probability that this individual has dengue, versus the posterior probability for individuals that test positive for dengue.

Challenging Problems

1. Three players A, B, and C take turns in order and independently flip a coin.The first player to obain heads wins. Assume that the order of flips is A, then B, and then C.
   (a) If the coin is fair, what is the probability that each player wins?

(b) If $P\{\text{heads}\} = p$, what is the probability that each player wins?

(c) Find the limit of these probabilities as $p \to 0$.

2. Casella & Berger, Exercise 1.28, page 40.