

Random Variables and Distributions

Homework 3

Problems

1. Flip a fair coin. If the coin lands tails, roll one die. If the coin lands heads, roll 2. Let X be the random variable that total number of pips shown.
 - (a) What values can x take?
 - (b) Give the mass function for X .
 - (c) Find $P\{\text{coin lands heads}|X = x\}$ for all possible values of x .
2. A bridge hand consists of 13 cards.
 - (a) Let X be the number of \spadesuit . Give the distribution function for X .
 - (b) Find $P\{X \leq 2\}$ and $P\{X > 6\}$.
 - (c) Simulate the number of \spadesuit in 10,000 bridge hands and comment on how the values in the simulation match the values in part (a). The commands `replicate` and `table` may be useful.
3. Verify that the following functions are cumulative distribution functions. As you do, provide the range of values for x and the density function.
 - (a) $F(x) = 1 - \exp(-(x/\beta)^\alpha)$.
 - (b) $F(x) = \frac{2}{\pi} \arcsin(\sqrt{x})$.

Challenging Problems

1. Six balls are randomly distributed into six urns. Let X_i be the number of cells having exactly i balls.
 - (a) Find the distribution of X_2 .
 - (b) Simulate this process 10,000 times and see if this matches your answer in (a).
2. Let X be a continuous random variable with cumulative distribution F_X and density f_X . Assume for some value x_0 , that $P\{X \leq x_0\} < 1$ and define $G(x) = P\{X \leq x|X > x_0\}$
 - (a) Show that G is a cumulative distribution function.
 - (b) Let Y be a random variable with cumulative distribution function G . Find the density function for Y in terms of F_x and f_X .