

Transformations and Expectations

Homework 4

Problems

- Two evenly matched teams are playing a best of 7 series, i.e., the teams play until one of them wins 4 games. Let X be a random variable that denotes the number of games in the series.
 - Give the mass function for X .
 - Find EX .
 - Draw the survival function for X and show that the area below the survival and above the horizontal axis equals EX .

- For $\beta > 0$, let X be a Pareto random variable with density

$$f_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{\beta}{x^{\beta+1}} & \text{if } x \geq 1 \end{cases}$$

- For $p > 0$, let $Y = X^p$. Find the density for Y .
- Use this to find $P\{Y > 2\}$.

- X have the Gumbel distribution,

$$F_X(x) = \exp(-e^{-x}).$$

- Find $P\{0 < X \leq 2\}$
- Find $f_X(x)$, the density of X .
- Display on a graph of the density the probability in part (a)
- Use the probability transform to create 1000 samples with this distribution.
- Estimate the probability in part (a) from this simulation.
- Find the first and third quartiles of X .
- Compare these values to the values in the simulation.

Challenging Problems

- For a binomial random variable, X , find $E(X)_2$.
- A person sits a restaurant table. The n -th customer chooses the first unoccupied table with probability $\alpha/(n-1+\alpha)$, and an occupied table with probability $c/(n-1+\alpha)$, where c is the number of people sitting at that table. Let X_n be the number of tables occupied after n customers have been seated. Find the distribution of X_4 and use this to find EX_4 .