Basics of Probability

Homework 1

Problems

1. The notation $\sigma(\mathcal{C})$ is used to denote the smallest σ-algebra generated by the collections of sets $\mathcal{C}$. Let A and B be subsets of a sample space Ω.
   (a) Find the number of distinct sets in $\sigma(\mathcal{C})$ where $\mathcal{C} = \{A, B\}$
   (b) List those sets.
2. Show that for 3 events, A_1, A_2, A_3,
   $$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2) \cap A_3)$$
3. The door on a a secure laboratory has a lock which has five buttons numbered from 1 to 5. The combination of numbers that opens the lock is a sequence of five numbers and is reset every week.
   (a) How many combinations are possible if every button must be used once?
   (b) Assume that the lock can also have combinations that require you to push two buttons simultaneously and then the other three one at a time. How many more combinations does this permit?

Challenging Problems

1. Assume that Ω is uncountable and let $\mathcal{F}$ be the collection of all subsets so that either A or A^c are countable.
   (a) Explain how $\mathcal{F}$ is a σ-algebra.
   (b) Define
   $$P(B) = \begin{cases} 0 & \text{if B is countable} \\ 1 & \text{if B is not countable} \end{cases}$$
   Show that P is a probability.
2. A telephone rings 10 times during the five-day work week. What is the probability of at least 1 call each day?