

# Random Variables and Distributions

## Homework 3

### Problems

1. Flip a biased coin with  $p$  the probability of heads. If the coin lands tails, roll a six-sided die. If the coin lands heads, a four-sided die. Let  $X$  be the random variable that total number of pips shown.
  - (a) What values can  $x$  take?
  - (b) If  $P\{X = 1\} = 1/5$ , find  $p$ .
  - (c) Give the mass function for  $X$ .
  - (d) Find  $P\{\text{coin lands heads}|X = x\}$  for all possible values of  $x$ .
2. A bridge hand consists of 13 cards.
  - (a) Let  $X$  be the number of  $\spadesuit$ . Give the distribution function for  $X$ .
  - (b) Find  $P\{X \leq 2\}$  and  $P\{X > 6\}$ .
  - (c) Simulate the number of  $\spadesuit$  in 10,000 bridge hands and comment on how the values in the simulation match the values in part (a). The commands `replicate` and `table` may be useful.
3. Verify that the following functions are cumulative distribution functions. As you do, provide the range of values for  $x$  and the density function.
  - (a)  $F(x) = 1 - \exp(-(x/\beta)^\alpha)$ ,  $\alpha > 0, \beta > 0$ .
  - (b)  $F(x) = \frac{2}{\pi} \arcsin(\sqrt{x})$ .

### Challenging Problems

1. Six balls are randomly distributed into six urns. Let  $X_i$  be the number of cells having exactly  $i$  balls.
  - (a) Find the distribution of  $X_2$ .
  - (b) Simulate this process 10,000 times and see if this matches your answer in (a).
2. Let  $X$  be a continuous random variable with cumulative distribution  $F_X$  and density  $f_X$ . Assume for some value  $x_0$ , that  $P\{X \leq x_0\} < 1$  and define  $G(x) = P\{X \leq x|X > x_0\}$ 
  - (a) Show that  $G$  is a cumulative distribution function.
  - (b) Let  $Y$  be a random variable with cumulative distribution function  $G$ . Find the density function for  $Y$  in terms of  $F_x$  and  $f_X$ .