Transformations and Expectations

Homework 4

Problems

1. Two evenly matched teams are playing a best of 7 series, i.e., the teams play until one of them wins 4 games. Let X be a random variable that denotes the number of games in the series.
   (a) Give the mass function for X.
   (b) Find EX.
   (c) Draw the survival function for X and show that the area below the survival and above the horizontal axis equals EX.

2. For $\beta > 0$, let X be a Pareto random variable with density

$$f_X(x) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{\beta}{x^{\beta+1}} & \text{if } x \geq 1 \end{cases}$$

   (a) For $p > 0$, let $Y = X^p$. Find the density for Y.
   (b) Use this to find $P\{Y > 2\}$.

3. X have the Gumbel distribution,

$$F_X(x) = \exp(-e^{-x}).$$

   (a) Find $P\{0 < X \leq 2\}$
   (b) Find $f_X(x)$, the density of X.
   (c) Display on a graph of the density the probability in part (a)
   (d) Use the probability transform to create 1000 samples with this distribution.
   (e) Estimate the probability in part (a) from this simulation.
   (f) Find the first and third quartiles of X.
   (g) Compare these values to the values in the simulation.

Challenging Problems

1. For a binomial random variable, X, find $E(X)_2$.

2. A person sits a restaurant table. The n-th customer chooses the first unoccupied table with probability $\alpha/(n-1+\alpha)$, and an occupied table with probability $c/(n-1+\alpha)$, where c is the number of people sitting at that table. Let X_n be the number of tables occupied after n customers have been seated. Find the distribution of X_4 and use this to find EX_4.