

Transformations and Expectations

Homework 5

Problems

1. Let Z be a standard normal random variable. Find $E|Z|$ and $E|Z^3|$
2. Let U be uniform on $[0, 1]$ and define $Y = \tan\left(\frac{\pi U}{2}\right)$
 - (a) Find the density of Y .
 - (b) Explain why Y does not have a mean.
 - (c) Simulate 10,000 values of Y and give the top 5 values in your simulation.
3. Let X have density

$$f_X(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ \beta e^{-\beta x} & \text{if } x > 0. \end{cases}$$

- (a) Find m , the median of X .
- (b) Give the value of $E|X - m|$.

Challenging Problems

1. Let X be a continuous random variable. Show that

$$\min_a E|X - a| = E|X - m|$$

where m is the median of X

2. The entropy of a discrete random variable X with mass function $f_X(x)$ is

$$H(X) = -E[\ln f_X(X)] = -\sum_x f_X(x) \ln f_X(x).$$

Find the entropy of

- (a) a Bernoulli random variable,
- (b) a uniform random variable, and
- (c) a geometric random variable.