

Transformations and Expectations

Homework 6

Problems

1. For $\beta > 0$, let X have density

$$f_X(x|\beta) = \begin{cases} \frac{4x^2}{\beta^3\sqrt{\pi}} \exp\left(-\frac{x^2}{\beta^2}\right) & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

- (a) Verify that $f_X(x|\beta)$ is a density.
 - (b) Find the mean and variance of X .
2. Using the density function directly, find the mean and variance of a $Beta(\alpha, \beta)$ random variable.
 3. Find the skewness of a $\Gamma(\alpha, \beta)$ random variable.

Challenging Problems

1.
 - (a) Using the mass function directly, find the probability generating function of a $Pois(\lambda)$ random variable.
 - (b) Let $p = \lambda/n$, show that the probability generating function of a $Bin(n, \lambda/n)$ random variable convergence to the probability generating function of a $Pois(\lambda)$ random variable.
2. For $\beta > 0$, let X have the Laplace density

$$f_X(x) = \frac{1}{2\beta} \exp\left(-\frac{|x|}{\beta}\right).$$

- (a) Find the standard deviation of X .
- (b) Place the standardized versions of X and a standard normal on the same graph and compare them.
- (c) Find the kurtosis of the Laplace distribution