Families of Random Variables

Homework 7

Problems

1. Let $X \sim Negbin(4, 2/5)$.
   (a) Find $P\{X > 10\}$
   (b) Find the smallest value for x so that $P\{X > x\} = 0.05, 0.01, 0.001$
   (c) Provide 10,000 simulations of the sum of four $Geom(2/5)$ random variables, check
      i. that the simulated mean and variance is close to that of a $Negbin(4, 2/5)$ random variable.
      ii. the answers in (a) and (b).

2. For some natural numbered-valued random variables, the case $\{X = 0\}$ cannot be observed. Thus, we can define and second random variable Y so that

$$P\{Y = y\} = P\{X = y | X \neq 0\}.$$

   (a) Find the mass function for Y.
   (b) Give this mass function for $X \sim Bin(n, p)$
   (c) Give this mass function for $X \sim Geom(p)$
   (d) Find the mean for these random variables,

3. On the other hand, the mass function for a zero inflated random variable Y is a mixture of a degenerate random variable taking only the value 0. Thus, the mass function

$$f_Y(y) = \alpha I_0(y) + (1 - \alpha) f_X(y)$$

   for some natural numbered-valued random variable X (mean μ_X and variance σ_X^2) and some $\alpha \in (0, 1)$
   (a) Verify that f_Y is a mass function.
   (b) Find the mean of Y in terms of μ_X and σ_X^2.
   (c) Find the variance of Y in terms of μ_X and σ_X^2.

Challenging Problems

1. Let $X \sim Beta(\alpha, \beta)$
   (a) Write the density of X as an exponential family with natural parameters η_1, η_2.
   (b) Find $E \ln X$

(c) Find $EX \ln X$

(d) What does these values tell you in the case of $X \sim Beta(1,1) = Unif(0,1)$?

2. Let $X_1, \ldots, X_n$ be independent and identically distributed with kurtosis α_4.

   (a) Find the kurtosis $\alpha_{4,n}$ of $\bar{X}_n$, the average of $X_1, \ldots, X_n$.

   (b) Find

$$\lim_{n \to \infty} \alpha_{4,n}.$$