Conditional Distributions and Transformations

Homework 9

Problems

1. Let X_1 and X_2 be independent uniform on the consecutive integers $-n, -(n+1), \ldots, n-1, n$. Use convolution to find the mass function for $X_1 + X_2$.

2. Let Z_1 and Z_2 be independent $N(0,1)$ random variables.

   (a) Find the density of Z_1^2.

   (b) Find the density of $Z_1^2 + Z_2^2$.

   (c) Show that $Z_1 + Z_2$ and $Z_1 - Z_2$ are independent and describe their distribution.

3. Use the Box Muller transform and the probability transform to simulate 100 pairs of standard normal random variables with correlations $\rho = -2/3, -1/3, 0, 1/3, 2/3, 1$. Display each simulation in a separate scatterplot.

Challenging Problems

1. Let X_i, $i = 1, 2, 3$ be independent $\Gamma(\alpha, 1)$ random variables.

   (a) Find the density of $S = X_1 + X_2 + X_3$

   (b) Find the joint density of $Y_i = X_i/S$, $i = 1, 2$.

   (c) Find the density of $Y_1 + Y_2$

2. Let $X \sim \Gamma(\alpha, \beta)$ and $Y|X \sim Pois(X)$

   (a) Find $E[Y|X]$ and $\mathrm{Var}(Y|X)$.

   (b) Find $\mathrm{Var}(Y)$.

   (c) Find the density of Y. What family of random variables contains Y.