Sufficiency

Homework 2

Problems

1. Find a sufficient statistic for a $Beta(\alpha, \alpha)$ distribution.
2. Let $X_1, \cdots, X_n \sim N(\mu_0, \sigma^2),\ \mu_0$ known, be independent Show that

$$\sum_{i=1}^{n}(X_i - \mu_0)^2.$$

   is minimal sufficient.

3. For a location-scale family, show that the sample skewness

$$S(\mathbf{X}) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{(\sum_{i=1}^{n}(X_i - \bar{X})^2)^{3/2}}$$

   is an ancillary statistic.

Challenging Problems

1. The a Pareto density, $Pareto(\alpha_0, \beta)$, with parameters $\alpha_0 > 0, \beta > 0$

$$f_X(x|\alpha_0, \beta) = \beta\alpha_0^{\beta}x^{-(\beta+1)}, \quad x > \alpha_0.$$

   (a) For an independent sample of n $Pareto(\alpha_0, \beta)$ random variables, assuming α_0 is known, find a sufficient statistic for β using the factorization theorem.

   (b) Find a sufficient statistic writing the density in the form of an exponential family.

   (c) Show that it is a minimal statistic.

   (d) If α_0 is unknown, is $Pareto(\alpha_0, \beta)$ an exponential family? Explain you answer.

   (e) Find a sufficient statistic for an independent sample of n $Pareto(\alpha_0, \beta)$ random variables, assuming α_0 is unknown.

2. For $X_1, \cdots, X_n \sim N(\theta, \theta^2)$, $T(\mathbf{X}) = (\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2)$ is sufficient but not complete.