Point Estimation

Homework 4

Problems

1. Chromosome 1 is the largest human chromosome spanning about 249 million nucleotide base pairs with an estimated 4,316 genes. The number of recombination events on a chromosome due to a single meiosis is modeled as a Poisson random variable. Its mass function is

$$f(x|\lambda) = \frac{\lambda^x}{x!}e^{-\lambda}.$$

   This parameter λ is called the *genetic length* of a chromosome with units in Morgans.

   (a) Let $X_1, \cdots X_n$ be the number of recombination events on n independent copies of human chromosome 1. Give the likelihood function $L(\lambda|x_1, \cdots x_n)$.

   (b) Find a sufficient statistic for this likelihood.

   (c) Find the maximum likelihood estimate $\hat{\lambda}$. Is it unbiased?

   (d) What is the variance of this estimator?

   (e) Find the Fisher information for the $Pois(\lambda)$. Is the variance for $\hat{\lambda}$ lowest possible?

   (f) Evaluate the estimate on the data $x_1 = 5, x_2 = 2, x_3 = 3, x_4 = 3, x_5 = 0, x_6 = 2, , x_7 = 0, , x_8 = 6, , x_9 = 4$, and $x_{10} = 3$.

2. (a) Let $T \sim Pois(\theta)$. Find $E_\theta e^{sT}$.

   (b) Let $X_1, \cdots X_n \sim Pois(\lambda)$. Find a UMVUE for e^λ.

Challenging Problems

1. Let $X_1, \ldots, X_n$ be n independent measurements of some unknown value μ. The likelihood function for the i-th measurement is

$$f_{X_i}(x_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp -(x_i - \mu)^2/2\sigma_i^2.$$

   (a) Give the likelihood for $X_1, \ldots, X_n$.

   (b) Find the maximum likelihood estimator $\hat{\mu}$ for μ.

   (c) $\hat{\mu}$ is the weighted average of the observations $x_1, \ldots, x_n$. What are the weights?

   (d) Find the variance of the estimator $\hat{\mu}$.

   (e) Explain to a lab assistant using non-technical language why you are using the weighted average above.

2. For a parameter $\theta > 0$, we model the accuracy of a dart player in hitting the center of the dartboard by the density

$$f_X(x|\theta) = \begin{cases} 0 & \text{if } x < 0, \\ \theta x^{\theta-1} & \text{if } 0 \leq x < 1, \\ 0 & \text{if } 1 \leq x, \end{cases}$$

for a continuous random variable X, the distance the dart is from the center of the board..

(a) Sketch the density for the values $\theta = 1/2$ and $\theta = 2$.

(b) Which one is the density of the more accurate dart player? Explain your answer

(c) Find the mean $\mu = E_\theta X$.

(d) For independent observations, $x_1, x_2, \ldots, x_n$ of this random variable from a given value of θ, give $\hat{\theta}$, the method of moments estimate.

(e) Find the variance of this estimator.

(f) Find the maximum likelihood estimator.

(g) Find its variance and compare this with the method of moments estimator.